Implications of Worker Classification in On-Demand Economy

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How should workers in the on-demand economy be classified? As contractors, employees, or somewhere in between? We study this policy question focusing primarily on the welfare of full-time workers, who have worked as much as employees but have been treated as contractors. We develop a game-theoretic queueing model with a service platform and two types of workers: full-timers who may choose gig jobs as primary income sources and commit to high availability for the platform and part-timers who do gigs for supplemental incomes and have only limited availability. We show that in the status quo of contractor mode, a company would efficiently differentiate workers' earnings in peak and off-peak periods to make full-timers commit upfront (*temporal incentive pooling*). While part-timers serve as a useful capacity recourse on the spot, to incentivize their participation the company may trade off the efficiency brought by the temporal incentive pooling, yet this in turn can create full-timers a positive surplus. As such, when all gig workers are reclassified as employees (according to, e.g., the California Assembly Bill No. 5) and part-timers exit the market, full-timers can be *undercut* (underpaid or underhired) by the profit-maximizing company and end up with lower welfare. When all are reclassified as "contractors$^+$," a UK practice that provides incomplete employee benefits but allows workers to self-join, workers may *overjoin* such that full-timers' utilization rate can remain low and their welfare not effectively enhanced. In light of these issues, we consider a differentiated scheme that classifies only full-timers as employees and treats part-timers as contractors. This hybrid mode still suffers from undercutting but curbs overjoining; it may also do less harm to consumers and the platform operator than uniform classifications. We also study a differentiated dispatch policy that prioritizes full-timers over part-timers. We demonstrate the potential of this operational approach to counteract both undercutting and overjoining. Finally, we calibrate the model and apply our insights to the ride-hailing market in California.

## 1. Introduction

Online platforms have created such disruptive impacts worldwide over the last decade. Perhaps most notably, with their well-designed mobile apps and advanced information technology, platforms like Uber completely revolutionize the way the labor force is organized (Tomassetti 2016). Workers are now given the opportunity to decide by themselves where, when, and how long to work, and they can switch from one platform (e.g., Uber) to another (e.g., Lyft) for better pay at any time rather than adhering to a nine-to-five schedule. Because of such flexibilities, platform companies have long treated workers as *independent contractors* (contractors for short) instead of *employees* (Katz 2015). The key issue with the contractor classification is, however, that workers will not be entitled to employee benefits such as a minimum wage guarantee and unemployment insurance (Radia 2019).[1] While this might not be a big concern for *part-time* workers, who make up 52–66% of the workforce and do "gigs" mostly to supplement the income from their full-time jobs, the adverse impacts can be substantial for *full-time* workers, who take gig jobs as primary income sources and have worked as much as full-time employees (Hall and Krueger 2018, Mishel 2018). It has been reported that after deducting vehicle expenses and self-funded insurance and social security, U.S. Uber drivers in 2018 earned on average $11.77 an hour, whereas the "average hourly compensation of workers in the lowest-paid major occupation" is $14.99 (Mishel 2018). Evidence also shows that gig companies depend on full-timers for a critical portion of their businesses (Parrott and Reich 2020). In other words, full-timers may have been largely exploited.

Regulators around the globe have been attempting to reclassify gig workers so as to enhance their welfare. Some pieces of legislation, such as the California Assembly Bill No. 5 (AB5), have ruled that workers on platforms like Uber must be treated as employees (Lazo 2019). Gig workers covered by this legislation must be provided with complete employee benefits; in particular, they must be compensated both when working (i.e., utilized) and when waiting for new "gigs" (i.e., idle),[2] which can dramatically increase labor costs and has thus aroused strong reactions from gig companies.[3] Alternatively, the Supreme Court of the United Kingdom ruled that Uber drivers must be treated as so-called *contractors*$^+$, an intermediate status between contractors and employees, with which Uber has complied (Schechner and Olson 2021). The ruling requires less workers' compensation than AB5: the company only needs to provide workers with some benefits, such as the minimum wage and holiday pay, and need not compensate workers for their idle time.

The implications of these rulings, especially for the welfare of full-timers, have been unclear. In the employee mode (EM), though workers will be compensated both when they are utilized and when they are idle, companies are likely to tighten their control over workers (e.g., set regular shifts and prohibit multihoming) in order to curtail labor costs and avoid unnecessarily paying for

[1] The right to unionize is also critical; see, e.g., `https://www.wsj.com/articles/uber-lyft-and-others-launch-campaign-to-head-off-unions-11646733600`.

[2] Because workers are "engaged to wait." See U.S. Department of Labor (2008).

[3] Uber, Lyft, and DoorDash funded a ballot initiative campaign (Ballotpedia 2020) and eventually sought exemption from AB5.

workers' idle time.[4] In response, most part-timers, who treasure the flexibility to earn supplemental income outside their full-time jobs, will likely quit the market. Yet how are market outcomes affected by the presence of part-timers? Can full-timers as a whole be better off with complete employee benefits on the one hand but stronger control assumed by gig companies on the other? Then in the contractor$^+$ mode (C$^+$M), full-timers will receive fewer benefits than in the EM, but at the same time, companies will also assume less control as workers need not be compensated for idle time,[5] and thus part-timers may again participate. Will full-timers be better off in the C$^+$M?

To answer these questions, we consider the daily operations of an on-demand service platform, which is modeled as a queueing network. There are two types of workers: full-time and part-time. In the status quo, all workers are classified as contractors. In each time period during a day, workers who participate on the platform will first wait in a virtual queue for consumer requests. Requests randomly pop up and are assigned to workers on a first-come-first-serve (FCFS) basis or lost if there is no worker available. Workers start to work (i.e., be en route or serve) as soon as they are dispatched; after each service, they collect a piece-rate wage from the company and then rejoin the virtual queue to be dispatched again, on and on until the next period has commenced.

Workers differ in their availabilities for the platform. Full-timers who commit to gig jobs as the primary income source will be generally available, and they decide *upfront* whether to commit based on the expected long-term utility on the platform. Part-timers, in contrast, are mostly occupied by their primary jobs elsewhere and only have limited availability. In each time period, available workers will participate if the average earning rate on the platform exceeds their opportunity costs.

| | Compensation | Flexibility | Workforce |
|---|---|---|---|
| **Uniform Classifications** | | | |
| Contractor (CM) | Piece-rate wage | $\surd$ | Both Full- and Part-timers |
| Employee (EM) | Hourly wage + benefits** | $\times$ | Only Full-timers |
| Contractor$^+$ (C$^+$M) | Piece-rate wage + benefits* | $\surd$ | Both Full- and Part-timers |
| **Differentiated Classification** | | | |
| Hybrid (HM) | Full-timers: Hourly wage + benefits** | $\times$ | Both Full- and Part-timers |
| | Pull-timers: Piece-rate wage | $\surd$ | |

* Benefits will be provided only when fulfilling service requests.
** Benefits provided both when fulfilling and waiting for service requests.

**Table 1 Worker Classifications in On-Demand Economy**

To model the EM, we make the following changes: (i) the company will control the workforce size and set work schedule; (ii) part-timers will quit the market, yet full-timers can still be enrolled; and (iii) full-timers hired by the company will receive *complete* employee benefits. The model for the C$^+$M will be the same as the contractor mode (CM) except that a wage floor is in order so that workers will receive *prorated* benefits when utilized. Table 1 summarizes the modeling details.

[4] https://www.law.com/therecorder/2019/03/26/finding-a-third-path-in-bridging-the-employeecontractor-divide/.

[5] In fact, Uber still gives drivers in the UK the flexibility to self-join the platform (Schechner and Olson 2021).

We refer to the EM and C$^+$M as *uniform classifications* because all workers are homogeneously reclassified, despite the fact that full-timers need employee benefits more, while part-timers treasure flexibility. Our primary focus is on full-timers' welfare, measured by their expected surplus. For completeness, we also consider the implications for part-timers, consumers, and the company.

As an overview, in the EM, if the lump-sum employee benefit is either low or high enough, full-timers' welfare can be lower than in the CM. We term this the *undercutting* issue. To intuit, in the CM, the company would efficiently differentiate the earning rates over time (i.e., high earning rate only in peak periods) to make full-timers commit to the platform upfront (*temporal incentive pooling*). To further enroll part-timers, though, the company may offer even higher earning rates throughout the day, leaving full-timers a positive surplus. Now in the EM, the company will strictly control workers' schedules and part-timers exit the market. Though full-timers hired by the company will be compensated with complete benefits, when the lump-sum benefit is too low, full-timers can be underpaid and become worse off than in the CM where part-timers generate them a positive surplus. If, instead, the benefit is too high, the company will only hire a small number of full-timers and lay off the rest. Full-timers' welfare will again be lower than in the CM.

In the C$^+$M, full-timers' welfare may not be effectively enhanced either, and we term the issue here *overjoining*. On the one hand, the piece-rate wages in the CM must be high enough to incentivize both full- and part-timers who may have already overjoined; as a result, the wage floor in the C$^+$M may even be lower than current wages. On the other hand, when the wage floor does result in higher wages, though, more full- and part-timers will be attracted to the platform and cannibalize the time full-timers fulfill services and earn the increased wages. In fact, drivers in the UK expressed concerns that the C$^+$M "potentially increasing the amount of time many spend searching for passengers and decreasing their overall pay even more" (Schechner and Olson 2021).

In light of these drawbacks of uniform classifications, we propose two differentiated schemes. We first consider a hybrid classification in which full-timers are treated as employees while part-timers are treated as contractors. We show that full-timers will earn as much as in the EM, which implies that the hybrid mode (HM) still suffers from the undercutting in the EM, but curbs the overjoining in the C$^+$M. In addition, the HM will do less harm to consumers and the company than the EM; in some cases, the HM can even make these stakeholders better off than the CM. Second, we take an operational perspective and study the scheme that prioritizes full-timers over part-timers in the virtual queue for requests. We demonstrate that this priority scheme could simultaneously counteract undercutting and overjoining and also make other stakeholders better off than the CM.

In summary, our work sheds light on the issue of worker classification in the on-demand economy. We highlight that different types of gig workers have very different motivations and needs—full-timers rely on gigs as their primary source of income and need employee benefits much, while part-timers do "gigs" for supplemental income and value the flexibility of gig jobs more. We show that uniform classifications will not always make full-timers better off than in the current contractor mode. Differentiated schemes can help counteract issues with uniform classifications while also moderating the negative impacts on other stakeholders. We provide insights into when and why full-timers may (not) be better off being reclassified and, at the same time, offer regulatory prescriptions for potentially more effective classification schemes in the on-demand economy.

## 2. Literature Review

The paper relates to the recent literature on enhancing the welfare of disadvantaged workers in the gig economy. Hall et al. (2021) theoretically show that Uber drivers will not benefit from piece-rate wage increases because their utilization rate will fall over time. Asadpour et al. (2020) show that a company's labor cost may grow unbounded if a wage regulation accounts for workers' utilization rate, and it can be necessary for the company to limit the workers' flexibility in work schedules in order to sustain its business. Benjaafar et al. (2022) find that the average labor welfare will first increase and then decrease in the labor pool size. Yu et al. (2020) also study labor pool size regulation, and they show that such regulation can strike a balance between gig workers' earnings and the interests of other stakeholders. Tang et al. (2021) investigate operational strategies that increase the participation rate and welfare of safety-concerned female drivers. Nikzad (2020) finds that workers will be better off when there are multiple competing platforms than when there is only a single platform, while Benjaafar et al. (2020) find that workers who multihome on different platforms can be hurt by the platform competition, as both platforms may pay low wages to prevent their counterparts from freeriding the labor supply they have costly incentivized. Siddiq and Taylor (2021) show that workers will be worse off as ride-hailing platforms use autonomous vehicles to serve riders. Krishnan et al. (2022) show that letting workers opt into a priority dispatching mode can both enhance workers' welfare and increase a gig company's profit. While some papers in this stream (e.g., Asadpour et al. 2020) have touched on the classification issue, little research has formally looked at whether gig workers can really be better off in classifications other than the status quo of CM. The work closest to ours is Hagiu and Wright (2019). The authors consider the interaction between a gig company and a single worker. We instead develop a framework with a random inflow of heterogeneous workers and focus on the welfare of those disadvantaged full-timers.

Our work also relates to the literature on platform operations with self-scheduling workers. Researchers have studied the capacity management (e.g., Afeche et al. 2018, Gurvich et al. 2019), pricing (e.g., Cachon et al. 2017, Taylor 2018, Banerjee and Johari 2019, Cachon et al. 2021), matching (e.g., Chu et al. 2018, Castro et al. 2021, Daniels and Turcic 2021, Feng et al. 2021, Hu and Zhou 2022), as well as synergies (Lian et al. 2021) and competition among companies (e.g., Bernstein et al. 2021). See Benjaafar and Hu (2020) and Hu (2021) for comprehensive reviews.

In particular, there has recently been an *employees-vs.-contractors* debate in this stream of literature. Dong and Ibrahim (2020) develop a queueing model with employees as deterministic servers but contractors as servers who only randomly show up in the system. The authors use a fluid approximation to analyze when it is optimal for a gig company to hire only employees, only contractors, or both types of workers. Lobel et al. (2022) study a similar question using a newsvendor model. In their model, a monopoly faces multiple states of demand distributions; it will decide the number of employees to hire or the wage to enroll contractors before a demand scenario realizes. The authors show that compared to employees, contractors are a more flexible resource as their labor supply can be controlled via the utilization rate. Furthermore, Chakravarty (2021) and He and Goh (2021) examine how a company can ration market demand between employees and contractors. All these papers have taken the worker classification as given and focus on a company's optimal operational policies. We instead attempt to approach the question of how gig workers should be classified, with a primary interest in the welfare of full-timers.

While it is tempting to think of the full-timers in our work as the employees in the aforementioned papers, we shall note that this particular group of contractors function rather as "pseudo-employees": indeed, they may commit upfront to high availability for the company to deploy, yet they also have the flexibility to self-schedule; this second aspect implies that the company must provide sufficient participation incentives by carefully controlling full-timers' utilization rates on spot, similar to how part-time contractors are managed. We complement previous papers by showing that a hybrid system of contractors (i.e., part-timers) and "pseudo-employees" (i.e., full-timers) does not necessarily outperform a pure "pseudo-employees" system for the company. The intuition is that maintaining part-timers' participation incentives may compromise the company's capability of *temporal incentive pooling*, i.e., generating temporally differentiated earning rates to make full-timers commit upfront. Nevertheless, we demonstrate that it is exactly via restricting such a strategy of differentiating earning rates that part-timers make the company generate higher earnings and, ergo, create a positive externality on their full-time counterparts.

Methologically, this paper relates to the literature on queueing games (see Hassin and Haviv 2003 for a comprehensive survey). Naor (1969) and Edelson and Hilderbrand (1975) pioneered the literature by studying monopoly pricing in an observable and unobservable queue, respectively. Our paper essentially studies a multi-period version of Edelson and Hilderbrand (1975). We investigate how a company shall motivate some agents to become frequent customers (i.e., full-timers) upfront while also attracting other infrequent customers (i.e., part-timers) to patronize on the spot, and how market outcomes hinge on interactions among the service operator and various customers.

Finally, the debate on worker classification is reminiscent of the classic Theory of Firm (Tomassetti 2016). See Appendix OA1 for a discussion of relevant work in this impactful school of thought.

## 3. Model & Equilibrium

Consider the daily operations of a service platform by a gig company. A day is segmented into $T$ periods, each spanning, e.g., 1 hour. Each period $s$ is either a "peak" ($\mathsf{h}$) or an "off-peak" ($\mathsf{l}$) period. We use the subscript $t$ to denote the period type, and for conciseness by "period $t$" we refer to "a period of type $t$". Define $\beta_{\mathsf{h}} \in (0,1)$ and $\beta_{\mathsf{l}} \equiv 1-\beta_{\mathsf{h}}$ as the fraction of peak and off-peak periods in a day, respectively. We model the worker-consumer matching and service processes using a two-station Jackson network illustrated in Fig. 1. There are two types of workers available for the platform, *full-time* and *part-time*, and the respective labor pool sizes are $M_f$ and $M_a$.[6] Full-timers are available during both peak and off-peak periods. Part-timers, in contrast, will be occupied by their primary jobs elsewhere most of the time. We assume that any part-timer will be available either only during peak or only during off-peak periods. Let $\gamma_{a,t}$ and $M_{a,t} = \gamma_{a,t} M_a$ denote the fraction and the pool size of part-timers available in some period $t \in \{\mathsf{h}, \mathsf{l}\}$, respectively.

The way we model full- and part-timers' availabilities is admittedly for smplicity; see Besbes et al. (2022b) for a formal treatment of workers' heterogeneity in availability during a day. Yet we have captured parsimoniously that full-timers are more available than part-timers, and this is consistent with the fact that a full-timer would work much more time than a part-timer (Mishel 2018). Also note that, in reality a part-timer may not be available for the platform during the same period of time (e.g., peak periods) everyday consistently even for a week, while full-timers can stick to a platform with high availabilities in the long term (e.g., two years; see Hall and Krueger 2018).

At the beginning of each period, available workers decide whether to join the platform, and we will detail this decision below. Those who join will first enter and wait in the virtual queue for

[6] We treat $M_f$ and $M_a$ as exogenous. According to Hall and Krueger (2018), full-timers tend to be younger people and those who are currently searching for a "steady" full-time job (rather than already holding one).

consumer requests. The time between any two consecutive requests is exponentially distributed with mean $1/\mu_t$ in period $t$. The demand rate is strictly higher in peak than in off-peak periods (i.e., $\mu_{\mathsf{h}} > \mu_{\mathsf{l}}$). Requests will be assigned to workers in an FCFS manner—that is, new requests will be assigned to the worker waiting at the head of the virtual queue or, in the queueing nomenclature, the "customer" that is "in service"—and lost if there is no worker available. Workers start services immediately after getting a request, and service times are i.i.d. with mean $1/\tau$.[7] After each service, workers rejoin the virtual queue for new requests, on and on until the next period commences. If still available by then, workers will decide whether to keep participating. For each service, the company collects from consumers a revenue $p_t > 0$. One may interpret $p_t$ as the expected price for a service that hinges on the average service time, the market demand, etc. In reality, the price for a service may vary according to its random duration. To keep the model parsimonious, we take $p_t$ as fixed in the baseline, and assume the price is strictly higher in peak than in off-peak periods (i.e., $p_{\mathsf{h}} > p_{\mathsf{l}}$). See Section 7.2.1 for an extension with endogenous service prices. Finally, workers are rewarded either on a per-piece or an hourly basis, depending on how they are classified.

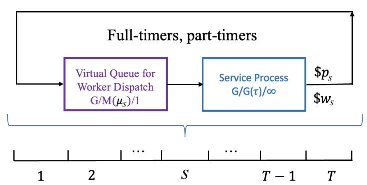



**Figure 1 Platform as a Queueing Network**

A few modeling details merit further clarification. First, we have abstracted away within-period market fluctuations (e.g., heavy rain around 3:30 PM that stimulates the demand for Uber). See Garg and Nazerzadeh (2022) for analysis on stochastic transitions between different market scenarios in relatively short timeframes. Also, we have assumed that workers will stay for the whole time period once they choose to participate, and within that period, they will always accept the gigs the company dispatches them to. In practice, workers may, however, stop at any moment within a period (see, e.g., Krishnan et al. 2022 for a model with workers' random exits after each service) and log in again later, and they may reject dispatches if more profitable gigs are anticipated (see, e.g., Lian et al. 2021, Garg and Nazerzadeh 2022). Our model is yet generally consistent with the recent structural econometric framework on gig workers' self-scheduling (e.g., Chen et al. 2020a).

Below we will study market outcomes in three different classification schemes: the contractor mode, the employee mode, and the contractor$^+$ mode. We adopt the notion of subgame perfect

[7] For simplicity, we normalize the service rates across different time periods to be homogeneous (i.e., $1/\tau_{\mathsf{h}} = 1/\tau_{\mathsf{l}} = 1/\tau$).

Nash equilibrium: workers' participation decisions will be solved first, and then the company's profit will be maximized in light of workers' best responses. We focus on the following outcome measures: full-timers' and part-timers' welfare, the transaction volume, and the company's profit; for conciseness, we defer the analysis for part-timers to Section 7.1. Note that, in our queueing setting, the transaction volume reflects the *fill rate*, i.e., the extent to which consumers' requests can be promptly fulfilled. We thus use the transaction volume as a measure of consumer welfare.

### 3.1. Contractor Mode

The contractor mode (CM) is arguably the status quo of the on-demand economy. In the CM, in each period $t$, the company will pay workers a piece-rate wage $w_t$ for fulfilling a service without guaranteeing that workers can earn the minimum wage or receive any employee benefit over time. One shall think of $w_t$ as the expected wage for a service; in reality, the wage per service may also vary according to its random duration. We first analyze workers' joining decisions given $\mathbf{w} = (w_\mathsf{h}, w_\mathsf{l})$.

**Part-timers' Strategy.** Recall that part-timers are mostly occupied by their primary jobs and are assumed to be available for the platform in only one of the total $T$ periods. At the beginning of each period, available part-timers will participate if the average earning rate $r_t$ for that period exceeds their opportunity cost $C_a$, which measures how much a part-timer values his or her time (e.g., the utility for staying with the family, the earning rate on a competing platform) and is drawn from some distribution with CDF $G_a(\cdot)$.[8] For ease of exposition, we will have $G_a$ degenerate to a fixed $c_a$, a common approach in the ride-hailing literature (see, e.g., Cachon et al. 2017, Sec. 5, Lobel et al. 2022). We define the average earning rate $r_t$ as the ratio of the piece-rate wage $w_t$ to the average time it takes to get and fulfill a service request, i.e., $r_t \equiv w_t/(W_t + 1/\tau)$, where $W_t$ is the average wait time for a request. We will characterize $W_t$ after clarifying full-timers' behaviors.

Following the convention in the queueing-game literature (see, e.g., Edelson and Hilderbrand 1975), in any period $t$, we will consider the mixed strategy equilibrium among part-timers, i.e., each available part-timer will participate with some probability $q_{a,t} \in [0,1]$, which can also be regarded as the fraction of part-timers that will choose to participate on the platform.

**Full-timers' Strategy.** Full-timers are those who may consider choosing gig jobs as their primary income source. To decide whether to make such a commitment, they compare the expected long-run utility on the platform $u$ with their outside option $u_0$ in the labor market, which we assume is a traditional nine-to-five full-time job[9]. Again we adopt the mixed strategy equilibrium. Let $q \in [0,1]$

[8] For simplicity, we abstract away any fixed cost to start working and material expense for staying on the platform (e.g., fuel).

[9] With a multi-platform setup, this outside option could be the expected long-run utility on another platform.

denote the probability that any full-timer commits to the platform. For those $1-q$ of full-timers who pick the outside option, we assume they will not participate later similarly as part-timers.

To characterize $u$ and $u_0$, note that a key reason for people to become full-time gig workers is the flexibility in their work schedule. They can kickstart a shift should they expect a surge in wage (e.g., during a peak period) or stop working (e.g., in an off-peak period) whenever they find it not worthwhile continuing. To capture the value of such flexibility, we assume that in any period $t$, just as part-timers, committed full-timers will participate if and only if the average earning rate $r_t$ is higher than their opportunity cost $C_f$. We similarly let the CDF $G_f(\cdot)$ of $C_f$ degenerate to a fixed point $c_f$, and consider the mixed strategy equilibrium among full-timers. Let $q_{f,t}$ denote the probability with which committed full-timers participate. Then full-timers' period-$t$ payoff is $q_{f,t}(r_t - c_f)^+$, and we define the expected long-run utility on the platform as $u \equiv T\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t q_{f,t}(r_t - c_f)^+$.[10] In contrast, those who choose the outside option have to follow the schedule set by their employer. To capture the loss of flexibility, we specify the outside option $u_0 \equiv T(r_0 + b - c_f)$, where $r_0$ and $b$ are the average earning rate and the lump-sum employee benefit from that outside job offer, respectively. For simplicity, we normalize $b = 0$. The functional form $r_0 - c_f$ (instead of $(r_0 - c_f)^+$) inside $u_0$ reflects the disutility of following a fixed work shift over a day. We assume $r_0 > c_f$ and that $c_f$ is different from part-timers' opportunity cost $c_a$ (i.e., $c_f \neq c_a$).

To pin down full-timers' commitment probability $q$, their as well as part-timers' participation probabilities $q_{f,t}$ and $q_{a,t}$ in period $t \in \{\mathsf{h},\mathsf{l}\}$, it is necessary to specify the average wait time for a service request $W_t$. Since participating workers will circulate on the platform for the entire time period, our model is essentially a closed queueing network; equivalently, the virtual queue for consumer requests (see Fig. 1) is a "finite-source" queue with workers' arrival rate depending on the difference between the total of participating workers and the number of workers in the service process. The average wait time to be dispatched is essentially the average sojourn time in the finite-source queue, which lacks a closed form amenable to analysis (Gross and Harris 1985). To facilitate the analysis, we follow the literature (e.g., Cachon and Feldman 2011) and approximate the virtual queue as an infinite-source queue so that $W_t$ takes the form of the average sojourn time for an M/M/1 queue. In particular, in any period $t \in \{\mathsf{h},\mathsf{l}\}$, with $M_t \equiv q \cdot q_{f,t} \cdot M_f + q_{a,t}M_{a,t}$ workers on the platform, the average wait time $W_t$ is $1/(\mu_t - \lambda_t)$, where $\lambda_t \equiv \tau M_t$. This approximation is generally valid if the labor pool is sufficiently large and the service rate $\tau$ is much less than the rate of service request $\mu_t$, which roughly align with the reality during peak periods. See Section 7.2.2 for a robustness check with the exact closed queueing network analysis.

[10] In reality, full-timers are likely to care more about payoffs in longer timeframes (e.g., a month). We essentially assume that variations in daily payoffs are mild, and the long-run payoffs can thus be reduced to the expected daily utility $u$ as defined.

Note though the average wait time $W_t$, approximated or not, is increasing in the number of workers participating on the platform. That is, as more workers come to do "gigs", each of them waits more to be dispatched; equivalently, their *utilization rate* becomes lower. As such, given the same piece-rate wage $w_t$, workers' average earning rate $r_t$ will be lower as more of them participate.

Define $\mathcal{Q} \equiv (q, q_{f,\mathsf{h}}, q_{f,\mathsf{l}}, q_{a,\mathsf{h}}, q_{a,\mathsf{l}})$ as the outcome of worker participations and $\mathcal{R} \equiv (r_\mathsf{h}, r_\mathsf{l})$ as the average earning rates given the piece-rate wages $(w_\mathsf{h}, w_\mathsf{l})$. We term the tuple $(\mathcal{Q}, \mathcal{R})$ a *market equilibrium*. Lemma 1 shows that the company's piece-rate wages admit a unique market equilibrium.

Lemma 1. *Given any pair of piece-rate wages $(w_\mathsf{h}, w_\mathsf{l})$, there exists a unique $(\mathcal{Q}, \mathcal{R})$.*

**The Company's Decision.** As per Lemma 1, any wage pair $(w_\mathsf{h}, w_\mathsf{l})$ will induce unique transaction volume $\lambda_t(w_\mathsf{h}, w_\mathsf{l})$ in period $t \in \{\mathsf{h}, \mathsf{l}\}$. The company's profit maximization problem is then

$$\max_{w_\mathsf{h}, w_\mathsf{l}} \Pi = T \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t (p_t - w_t) \lambda_t (w_\mathsf{h}, w_\mathsf{l}). \tag{1}$$

Note again that for now, we will abstract away the company's optimization over the service price $p_t$ since we have fixed the demand rates $\mu_t$ and the prices $p_t$ to keep the model parsimonious.

Denote by $\mathbf{w}^* = (w_\mathsf{h}^*, w_\mathsf{l}^*)$ the optimal piece-rate wages to (1), by $\mathbf{r}^* = (r_\mathsf{h}^*, r_\mathsf{l}^*)$ the corresponding average earning rates in the peak and off-peak periods, and by $u^*$ the expected long-run utility for committed full-timers, respectively. Also denote by $\mathcal{Q}^*$ the equilibrium participation outcome among full- and part-timers, and by $S^*$ full-timers' welfare. We define $S^*$ as the expected surplus among full-timers who commit to the platform, i.e.,

$$S^* \equiv q^* M_f (u^* - u_0) = q^* M_f (T \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t q_{f,t}^* (r_t^* - c_f)^+ - u_0).$$

Further denote by $\lambda^*$ the transaction volume and by $\Pi^*$ the company's profit in equilibrium. We will study the CM equilibrium by comparing the market outcomes with and without part-timers to see how they affect the company's operations and, thus, full-timers' welfare. Our analysis here complements Cachon et al. (2017) by shedding light on the role of part-time service providers who would not consider joining the platform in the long run but may only participate in the short term. As we will show in Section 4, part-timers indeed form a driving force behind market outcomes in different classification schemes.

Proposition 1 (CM Equilibrium). *Let $S_{fo}^*$, $\lambda_{fo}^*$ and $\Pi_{fo}^*$ denote full-timers' welfare, the transaction volume and the company's profit in the full-timer-only CM equilibrium. We have:*

*(i) For full-timers, $S^* \geq S_{fo}^* = 0$. There exist $\overline{M}$, $\underline{c}$ and $\overline{c}$ such that if full-timers' labor pool $M_f \leq \overline{M}$ and part-timers' opportunity cost $c_a \in (\underline{c}, \overline{c})$, we have $S^* > 0$.*

*(ii) For consumers, there exists $\overline{c}'$ such that if $c_a \geq \overline{c}'$, $\lambda^* \geq \lambda^*_{fo}$. (iii) For the company, there exist $\underline{c}''$, $\overline{c}''$, and $\overline{c}'''$ such that (a) if $c_a \geq \overline{c}'''$, $\Pi^* \geq \Pi^*_{fo}$ and (b) if $c_a \in (\underline{c}'', \overline{c}'')$, $\Pi^* \leq \Pi^*_{fo}$.*

Proposition 1(i) says full-timers' welfare is higher in the CM equilibrium with part-timers than without. In particular, their welfare will be strictly higher (i.e., $S^* > S^*_{fo} = 0$) if their population is sufficiently low (i.e., $M_f \leq \overline{M}$) while part-timers' opportunity cost is moderately high (i.e., $c_a \in (\underline{c}, \overline{c})$). In other words, part-timers create a positive externality on full-timers.

To intuit, recall that full-timers will commit to the platform if the expected utility $u$ is (weakly) higher than the outside option $u_0$ (i.e., $u \geq u_0$). In the full-timer-only CM, clearly, the company will choose piece-rate wages $\mathbf{w}$ that are just enough to meet this incentive constraint (i.e., $u = u_0$) since even a strictly positive surplus at best attracts all full-timers available from the labor pool.

More importantly, rather than meet the incentive constraint $u = u_0$ with wages $\mathbf{w}$ that generate constant earning rates over time (i.e., $r_\mathsf{h} = r_\mathsf{l}$), the company can set intertemporally differentiated earning rates.[11,12] Indeed since the market demand is higher in peak than in off-peak periods, it is most efficient for the company to offer a high (low) earning rate in peak (off-peak) periods (i.e., $r_\mathsf{h} > r_\mathsf{l}$). We refer to this operational apparatus as the *temporal incentive pooling*. On the one hand, the high demand in peak periods generally ensures workers a higher utilization rate than in off-peak periods, and it is thus much easier for the company to generate the high earning rate $r_\mathsf{h}$ using a piece-rate wage in peak periods than in off-peak periods. On the other hand, since the off-peak market may not need all committed full-timers to serve, offering the low earning rate $r_\mathsf{l}$ in off-peak periods has the extra advantage of effectively moderating full-timers' participation rate.[13]

Now, in the market with part-timers, when full-timers' population is relatively small (i.e., $M_f \leq \overline{M}$), the company will also enroll part-timers to better serve the market, either in peak periods when the demand is surging or throughout the day. When their opportunity cost is sufficiently high (i.e., $c_a > \underline{c}$), even the high earning rate $r_\mathsf{h}$ under the temporal incentive pooling strategy fails to provide part-timers with enough incentives. As such, the company must pay higher piece-rate wages to generate higher earning rates than under the pooling strategy (i.e., $r'_t > r_t$ for some

[11] In fact, a common wisdom among full-time gig workers is to seize the "ultra" earning opportunities in peak hours; see, e.g., `https://therideshareguy.com/everything-i-wish-i-knew-before-becoming-a-full-time-uber-and-lyft-driver/`.

[12] When full-timers have heterogeneous outside options in the labor market, the company is expected to use the temporal incentive pooling to target the *marginal* full-timer whose outside option $u_0$ equals the platform utility $u$ and who thus ends up with no surplus. *Inframarginal* full-timers whose outside option $u_0 < u$ will have a positive surplus.

[13] In reality platforms may sometimes fix the "payout ratio" (i.e., $w_t/p_t = \xi$ for some constant $\xi$ for all $t$). This certainly limits the temporal incentive pooling, since the company may need to raise wages in some periods to align with the payout ratio in other periods. Yet higher wages also bring in more workers, and the earning rates in those periods will not necessarily increase. As such, full-timers may again end up with a meager surplus (i.e., $u = u_0$); that is, the company may still do the temporal incentive pooling under the fixed payout ratio.

$t \in \{\mathsf{h}, \mathsf{l}\}$), which in turn leaves full-timers a positive surplus (i.e., $u' > u_0$). This result complements the literature (e.g., Benjaafar et al. 2022) with a fresh insight into the relation between the labor pool size and welfare. We show that those who commit to high availability (i.e., full-timers) will be better off if those with limited availability (i.e., part-timers) participate (i.e., labor pool expansion).

While we have seen through Proposition 1(i) that part-timers' presence is a boon for full-timers, Proposition 1(iii) implies it could be a bane for the company: it says the company's optimal profit can actually be lower with part-timers than without (i.e., $\Pi^* \leq \Pi^*_{fo}$), and this happens when part-timers' opportunity cost $c_a$ is in an intermediate range (i.e., $(\underline{c}'', \overline{c}'')$). To understand this result, when the cost $c_a$ is either very low (i.e., $c_a \leq \underline{c}''$) or very high (i.e., $c_a \geq \overline{c}''$), part-timers have a sharp cost advantage or disadvantage over full-timers; the company can then make more profit than without their presence by either having them or full-timers play the major role in the workforce.

When $c_a$ is in the intermediate range, neither worker group has much cost advantage over the other. However, part-timers' participation can make it more costly for the company to engage full-timers to commit upfront. The intuition is that, according to the temporal incentive pooling strategy, the company offers a high earning rate $r_{\mathsf{h}}$ in peak periods. Because part-timers' opportunity cost $c_a$ is not very high (i.e., $c_a < \overline{c}''$), the earning rate $r_{\mathsf{h}}$ will attract many part-timers to participate, which lowers workers' utilization rates and thus lowers the earning rate to some $r''_{\mathsf{h}} < r_{\mathsf{h}}$. But then full-timers' expected utility drops to $u'' < u_0$, and to keep full-timers engaged, the company must raise the off-peak earning rate to some $r''_{\mathsf{l}} > r_{\mathsf{l}}$. Yet as our earlier discussion implies, the "repooled" earning schedule $(r''_{\mathsf{h}}, r''_{\mathsf{l}})$ will be less efficient than $(r_{\mathsf{h}}, r_{\mathsf{l}})$; in other words, part-timers restrict the company's capability of temporal incentive pooling to retain full-timers.

### 3.2. Employee Mode

We make the following assumptions on the company's operations in the EM.

First, we assume that the company will perform admission control of workers and set a fixed work schedule. This follows the high-level assumption that gig companies in the EM will impose strict workplace rules rather than give workers the same flexibility as in the CM. The purpose is to keep the utilization rate of workers at a reasonable level and avoid paying for workers' unprofitable activities. As one supporting example, Hermet, a UK delivery company, required drivers to follow pre-designed routes and schedules after it started to treat drivers like employees.[14] We also assume that every hired worker will serve both in peak and off-peak periods. Many relevant papers have adopted essentially the same assumption (e.g., Milner and Pinker 2001, Dong and Ibrahim 2020),

[14] https://www.hrmagazine.co.uk/content/features/uber-drivers-are-now-employees-but-is-it-a-victory-for-workers-rights.

implying that it founds a proper benchmark to compare the other classification schemes with. See Appendix OA3.4 for our discussions on the dynamic scheduling in the EM.

We next assume that part-timers will quit the market while full-timers will still consider joining the platform. Since full-timers already work as much as full-time employees, they may not mind trading off some flexibility for more benefits (Sonnemaker 2020). For part-timers, there is abundant evidence showing that they treasure the flexibility to earn supplemental income outside their full-time jobs. A delivery company for cannabis dispensaries in the United States reported that half the drivers left after they were reclassified as employees in 2018, mostly because they did not like the idea of becoming employees or having to work regular shifts.[15] Researchers have also empirically investigated the negative impacts of reduced schedule flexibility on part-timers' welfare (see the discussion in, e.g., Chen et al. 2019, Yu et al. 2022). We do acknowledge that there might be some part-timers staying on the platform as part-time employees and that some full-timers who highly value the flexibility might quit as well. Our key insights shall extend to those situations.[16]

Finally, we assume that for every time unit, whether a worker is waiting for a service request or fulfilling one, the company will pay (i) an hourly wage $r$, and (ii) a lump-sum benefit $B \geq 0$ that includes any other employee benefit such as unemployment insurance. We assume that the company can decide $r$ but will take $B$ as fixed. One can think of $B$ as a government mandate level or the average industry level of employee benefits. In addition, we assume that full-timers' outside earning rate $r_0$ is close to the social minimum wage, and the company must set $r \geq r_0$.

It is worthwhile to mention that New York City (NYC) has imposed an *earning rate* floor in the local ride-hailing market since 2018 (Asadpour et al. 2020). The employee reclassification nevertheless differs from the NYC regulation in several critical aspects. First, the NYC regulation measures workers' utilizations only every other six months, yet in the EM, companies must record the working hours on a real-time basis[17] and compensate workers even on days when the market tumbles. Second, the amount of employee benefits the NYC includes in the earning rate floor is much lower than the complete benefit level in the US service industry in general.[18] Finally, as employers, companies can exert far more control over workers than they could in NYC. Putting together, the EM is expected to create more profound impacts than the NYC regulation.

[15] https://www.theatlantic.com/technology/archive/2018/09/gig-economy-independent-contractors/570307/.

[16] One can think of the EM with dynamic scheduling that we study in Appendix OA3.4 as an approximation to such situations.

[17] See https://www.dol.gov/agencies/whd/fact-sheets/21-flsa-recordkeeping.

[18] In 2018, the amount of benefits mandated by the NYC is \$2.22/hour (Asadpour et al. 2020). The national average complete benefit in the service sector is over \$3.9/hour; in the transportation service sector, it is even over \$9.6/hour (Table 9 in https://www.bls.gov/web/ecec/ececqrtn.pdf). Workers' union right in the EM aggravates companies' overheads even further.

**Full-timers' Strategy.** Given our assumptions, the expected long-run utility from becoming an employee on the platform is $u_E = T((r+B)-c_f)$, where $r+B$ is the benefit-inclusive earning rate guaranteed by the company and $c_f$ reflects the disutility of following the company's schedule. Since their labor market outside option $u_0 = T(r_0 - c_f)$, full-timers would like to be hired for any $r \geq r_0$.

**The Company's Decision.** Since all full-timers would like to be hired for any $r \geq r_0$, it is optimal for the company to set $r^* = r_0$. The company then only needs to decide the most profitable size of the workforce. Define $r_B \equiv r_0 + B$ as the average earning rate for an employee, and $\lambda_f = M_f \tau$ as the gross size of the full-time workforce. The platform's profit maximization problem in the EM is

$$\max_{\lambda_E \leq \lambda_f} \Pi_E \equiv T \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t (p_t - r_B(\frac{1}{\mu_t - \lambda_E} + \frac{1}{\tau}))\lambda_E. \tag{2}$$

Denote by $\lambda_E^*$ the optimal workforce size, which also measures the transaction volume. Also, denote by $S_E^*$ full-timers' welfare and $\Pi_E^*$ the company's profit in the EM equilibrium. We define $S_E^* \equiv q_E^* M_f (u_E^* - u_0) = q_E^* M_f \cdot TB$, where $q_E^* = \lambda_E^* / \lambda_f$. Part-timers' welfare, in this case, is $\tilde{S}_E^* = 0$ as we assume they quit the market altogether. The EM equilibrium is characterized as follows.

Proposition 2 (EM Equilibrium). *In the equilibrium of the employee mode, we have*

*(i) There exists $B_E$ such that $S_E^*$ increases in $B$ on $[0, B_E]$ but decreases in $B$ for $B \geq B_E$.*

*(ii) The transaction volume $\lambda_E^*$ and the company's profit $\Pi_E^*$ both decrease in $B$.*

Proposition 2 says full-timers' welfare $S_E^*$ in the EM will first increase and then decrease in the lump-sum benefit $B$, while both the transaction volume and the company's profit will decrease in $B$. The properties of $\lambda_E^*$ and $\Pi_E^*$ are intuitive: as $B$ increases, the company's labor cost increases so that fewer full-timers will be hired and less profit can be made by the company. For full-timers, as $B$ increases, some of them will be laid off and lose $B$, while those who are still employed will be provided with a higher $B$. When $B$ is low, the company hires a large number of full-timers, and the fraction of layoffs is relatively low. Therefore, as $B$ marginally increases, the loss of the layoffs is less than the gain of the hires, and full-timers' welfare increases. When $B$ is sufficiently high, not many full-timers are still hired, and the fraction of layoffs becomes relatively large. As a result, the loss of the layoffs dominates the gain of the hires, and full-timers' welfare decreases in $B$.

### 3.3. Contractor$^+$ Mode

The UK contractor$^+$ mode (C$^+$M) is an intermediate classification between CM and EM: companies will provide workers with some employee benefits but not treat them exactly as employees (GOVUK 2021), while gig workers will have flexibility as contractors. In the case of Uber, the company guarantees that drivers earn the minimum wage and holiday pay, but only for the time they are fulfilling service requests; drivers can still set their own work schedules. Such flexibility implies that part-timers may again participate in the market. As such, C$^+$M is different from CM mainly in the sense that the piece-rate wages must be higher than $r_B/\tau$ so that workers receive prorated employee benefits at least when they are fulfilling services. The company's problem thus becomes

$$\max_{w_t \geq r_B/\tau} \Pi = T \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (p_t - w_t)\lambda_t(w_{\mathsf{h}}, w_{\mathsf{l}}). \tag{3}$$

Denote by $S_+^*$ full-timers' welfare, $\lambda_+^*$ the transaction volume and $\Pi_+^*$ the company's profit in equilibrium. To study the C$^+$M equilibrium, it is useful to take CM as a benchmark. In the next section, we will compare C$^+$M with CM to understand its impacts.

## 4. Implications of Uniform Classifications

In this section, we aim to understand how market outcomes, especially full-timers' welfare, will be affected if all gig workers are reclassified either as employees or as contractors$^+$.

### 4.1. Issues with Uniform Classifications: Undercutting & Overjoining

We first examine whether full-timers will always be better off as employees rather than as contractors. Combining Propositions 1(i) and 2, the following theorem summarizes our findings.

Theorem 1 (EM vs. CM for full-timers). *Comparing full-timers' welfare $S_E^*$ in the EM and $S^*$ in the CM, we have: (i) When $S^* = 0$, $S_E^* \geq S^*$. (ii) When $S^* > 0$, there exist $\underline{B}, \overline{B}$ such that $S_E^* \leq S^*$ if and only if $B \leq \underline{B}$ or $B \geq \overline{B}$.*

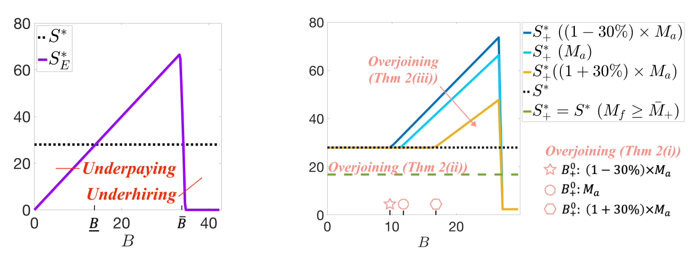
Theorem 1(i) shows that, if in the CM full-timers end up with a trivial surplus (i.e., $S^* = 0$, which happens if, e.g., only very few part-timers participate in the CM as Proposition 1 implies), they will indeed be better off as employees than as contractors (i.e., $S_E^* \geq S^*$). However, if full-timers in the CM already obtain a positive surplus, Theorem 1(ii) warns that they will be worse off as employees (i.e., $S_E^* \leq S^*$) if the lump-sum benefit is sufficiently low or high.

We term the issue in the EM *undercutting*. Recall that in the CM, part-timers' participation limits the company's capability of temporal incentive pooling and thus creates a positive externality on full-timers (Proposition 1(i)). Now in the EM, the profit-maximizing company strictly controls workers' schedules; part-timers quit the market due to the loss of flexibility, and so vanishes the positive externality on full-timers. Given that the company compensates each full-timer hired as an employee at the flat rate $r_B$, if the lump-sum benefit $B$ is too low (i.e., $B \leq \underline{B}$), full-timers will be *underpaid* and clearly not better off than in the CM. If $B$ is sufficiently high (i.e., $B \geq \overline{B}$), we have discussed in Proposition 2 that many full-timers will be laid off by the company (*underhiring*), and their losses of the benefit $B$ will render the full-timers' overall welfare once again lower than in the CM. We illustrate the negative impacts of underpaying and underhiring in Fig. 2(a).

Note, though, Benjaafar et al. (2022) show that gig workers will always be better off if regulators impose a floor for their average earning rates on the platform. Their finding, however, hinges on the assumption that workers are still allowed to join the platform freely. Our work instead stresses that the company can strictly control workers after regulators reclassify them as employees, and full-timers can be undercut by this fundamental change in the company's operations.

Hagiu and Wright (2019) find a similar issue in the EM. However, their work and ours differ on why a gig worker may earn more as a contractor. In Hagiu and Wright (2019), the company delegates the operation to and shares the final revenue with a contractor. In some cases, the contractor will choose the action that not only maximizes the company's profit but also leaves himself a surplus. We show instead that a full-time contractor can have a surplus thanks to the high earnings driven by part-time contractors' participation. Indeed, as we show in Proposition 1(i), without part-timers, a fill-timer will never receive a utility higher than their outside option, even if the company has delegated to him the pivotal decisions of when to participate.

Our results suggest that regulators should exercise caution on employee reclassification because whether full-timers will be better off hinges on their income levels as contractors. Researchers have not settled on how much gig workers earn currently. Using data from Uber, Hall and Krueger (2018) find that in a U.S. city such as San Francisco, after adjusting for expenses, an Uber driver's hourly income is higher than that of an employee taxi driver. In contrast, based on an online survey of 1,121 Uber and Lyft drivers in the U.S., Zoepf et al. (2018) find that 74% of drivers earn less than the minimum wage, and some of them even lose money after accounting for vehicle expenses. As such, it is imperative for regulators first accurately to measure full-timers' incomes as contractors.




(a) EM: undercutting (b) $C^+M$: overjoining

**Figure 2** Issues of Uniform Classifications

*Note.* $\beta_h = 0.2, M_f = 2, M_a = 34, \gamma_{a,h} = 0.8, \mu_h = 40, \mu_l = 25, \tau = 2, p_h = 45, p_l = 18, r_0 = 9, c_f = 8, c_a = 16.$

We now turn to the implications of $C^+M$.

Theorem 2 ($C^+M$ vs. CM for full-timers). *Comparing full-timers' welfare $S_+^*$ in the $C^+M$ and $S^*$ in the CM, there exist $\underline{B}_+, \overline{B}_+$ such that if $B \geq \overline{B}_+$, we have $S_+^* \leq S^*$. For $B \leq \underline{B}_+$, we have $S_+^* \geq S^*$ and (i) If $B \leq B_+^0 \equiv \min_{t\in\{h,l\}} \tau w_t^* - r_0$, we have $S_+^* = S^*$. (ii) If the labor pool size of full-timers $M_f \geq \overline{M}_+$ for some $\overline{M}_+$, we have $S_+^* = S^* = 0$. (iii) When $S^* > 0$, we have $(S_+^* - S^*)/S^*$ decreases in part-timers' labor pool size $M_a$.*

Recall that the $C^+M$ implements a wage floor $r_B/\tau$ to ensure that workers received *prorated* employee benefits for the time they fulfill services. According to Theorem 2, this wage floor can backfire and hurt full-timers if the benefit $B$ is sufficiently high (i.e., $B \geq \overline{B}_+$). The intuition is that the wage floor can be so high that the platform has to shut down in many periods over a day.

When $B$ is at a moderate level (i.e., $B \leq \underline{B}_+$), Theorem 2(i) says when the lump-sum benefit is too low (i.e., $B \leq B_+^0$), the wage floor in the $C^+M$ will not even enforce at the equilibrium piece-rate wages in the CM (i.e., $r_B/\tau \leq \min_{t\in\{\mathsf{h},\mathsf{l}\}} w_t^*$), and thus full-timers will certainly not be strictly better off than in the CM. Theorem 2(ii) says full-timers' welfare will never be enhanced should their labor pool size be sufficiently large. Furthermore, Theorem 2(iii) implies that the increase in full-timers' welfare will taper off in markets where there are potentially many part-timers.

We term the inefficiency here workers' *overjoining*, which is due to their flexibility to self-schedule both in the CM and $C^+M$. First, Theorem 2(i) pertains to workers' overjoining in the CM: the more workers joining, the longer the wait times and thus the higher the piece-rate wage $w_t^*$ in the CM to incentivize workers. As such, unless the benefit $B$ is sufficiently high, the wage floor $r_B/\tau$ will be too low to enforce and improve full-timers' welfare. This result complements Benjaafar et al. (2022) assume that the wage floor will always enforce in a similar setting.

Theorem 2 parts(ii) and (iii) are driven by workers' overjoining in the $C^+M$. Though workers are not overjoining until their own welfare becomes worse off than in the CM, their behaviors certainly slow down the process of enhancing full-timers' welfare. For Theorem 2(ii), because full-timers' labor pool size is sufficiently large (i.e., $M_f \geq \overline{M}_+$), the average wait time for a request will be too long (and thus the earning rates too low) if all of them join the platform. Hence, some full-timers will never commit nor participate, and the expected long-run utility on the platform $u$ must equal their outside option $u_0$. In the $C^+M$, at first, the wage floor raises the piece-rate wages, the earning rates, and thus the platform utility $u$. This attracts full-timers on the outside to the platform, ignoring that their participation will increase the average wait time and thus lower the utilization rate for all workers. In fact, new full-timers will keep joining until the expected utility $u$ falls back to $u_0$. Hence, full-timers will end up with no surplus for any $B$ (i.e., $S_+^* = S^* = 0$).

Finally, Theorem 2(iii) is related to part-timers' overjoining. At first, when the labor pool size $M_a$ is sufficiently low, not many of those who are occupied by their primary jobs elsewhere will be available at any time during the day, and as a result, the market will be mostly served—and thus the prorated benefits in the C$^+$M will be mostly harvested—by full-timers. As $M_a$ increases, more people are available on a part-time basis; as they enter the market, the relative amount of time full-timers fulfill service requests and earn the benefits is cannibalized. Hence, full-timers' welfare will increase on a much smaller scale than when there are fewer part-timers in the labor pool. We illustrate the inefficiencies associated with workers' overjoining in Fig. 2(b).

### 4.2. Implications for Consumers and the Company

We now briefly discuss how consumers and the company will be affected when gig workers are reclassified as employees or contractors$^+$. In the on-demand service setting, consumers' welfare hinges on two factors: the price and the fill rate. Intuitively, in uniform classifications, service prices will rise as gig companies' labor costs increase and thus hurt consumers. Given that, we will focus here on the *fill rate* measured by the daily average transaction volume $\lambda \equiv T \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t \lambda_t$.

Theorem 3 (Implications for Consumers). *For the transaction volume $\lambda^*_E$ in the EM, $\lambda^*_+$ in the C$^+$M, and $\lambda^*$ in the CM, we have (i) For $\overline{B}_+$ in Theorem 2 and some $B'_+ \leq \overline{B}_+$, we have $\lambda^*_+ \leq \lambda^*$ if $B \geq \overline{B}_+$ and $\lambda^*_+ \geq \lambda^*$ if $B \in [B'_+, \overline{B}_+]$. (ii) There exists $B'$ such that $\lambda^*_E \leq \lambda^*$ iff $B \geq B'$.*

Theorem 3 says compared with the CM, consumers can be worse off both in the EM and the C$^+$M when the lump-sum benefit $B$ is sufficiently high. In the EM this is consistent with the undercutting issue: given that part-timers will quit the market due to the loss of flexibility and the company will underhire full-timers when the benefit $B$ is sufficiently high, the fill rate will naturally be lower than in the CM. In the C$^+$M this is because the platform has to shut down in many periods due to the high wage floor $r_B/\tau$, as we have discussed in Theorem 2.

When $B$ is in an intermediate range (i.e., $B \in [B'_+, \overline{B}_+]$), consumers can become better off in the C$^+$M than in the CM. This result aligns with the overjoining issue we have identified: as the wage floor $r_B/\tau$ raise the piece-rate wages the company will pay, more full- and part-timers will be attracted to the platform and fulfill consumer requests than in the CM.

Theorem 4 (Implications for the Company). *Comparing the company's profit $\Pi^*_E$ in the EM, $\Pi^*_+$ in the C$^+$M, and $\Pi^*$ in the CM, we have $\Pi^* \geq \Pi^*_+$ and $\Pi^* \geq \Pi^*_E$. In addition, there exist $\underline{B}'$ and $\overline{B}'$ such that if $B \leq \underline{B}'$ or $B \geq \overline{B}'$, we have $\Pi^*_+ \geq \Pi^*_E$.*

Theorem 4 shows that the company will end up with the highest profit in a free market (i.e., CM). That the company's profit will be higher in the CM than in the C$^+$M (i.e., $\Pi^* \geq \Pi^*_+$) is straightforward, as the wage floor $r_B/\tau$ in the C$^+$M shrinks the set of feasible piece-rate wages.

To see why the company will make more profit in the CM than in the EM (i.e., $\Pi^* \geq \Pi^*_E$), on the one hand, unlike in the EM where only full-timers can be employed, in the CM, the company can enroll both full- and part-timers in a nimble fashion: it can make full-timers commit *upfront* as the base capacity, complement the base with part-timers in peak periods and shrink the base via a low earning in off-peak periods, or it can also "hybridize" the workforce all day long. One way or another, the company in the CM can scale the service capacity up to the level that is at least as efficient as in the EM in any period; we term this *dynamic capacity configuration*.[19]

On the other hand, while in the EM the company has to constantly guarantee minimum earnings and employee benefits, in the CM it can incentivize full-timers with temporally differentiated earning rates: it would offer a high earning rate in peak periods—the market demand and thus the service price are higher than in off-peak periods anyway—but provide a relatively low earning rate in off-peak time. Importantly, contingent on whether part-timers are enrolled, the company can fine-tune an earning rate schedule to both make full-timers commit upfront and incentivize part-timers on the spot, if at all. As we did in Section 3, we coin this the *temporal incentive pooling*.

Finally, to understand that the company will make more profit in the C$^+$M than in the EM for either sufficient low ($B \leq \underline{B}'$) or high ($B \geq \overline{B}'$) lump-sum benefit, when $B$ is sufficiently low, the wage floor in the C$^+$M will not enforce, and therefore the company will make the same amount of profit as in the CM. When the benefit $B$ becomes sufficiently high, since the company only has to afford workers' prorated employee benefits in the C$^+$M but is required to provide complete benefits in the EM, the adverse impact of the mounting labor cost is more substantial in the EM. Whether the C$^+$M will still ensure the company such a profit advantage over the EM for intermediate $B$ (i.e., $B \in [\underline{B}', \overline{B}']$), however, is unclear: on the one hand, the negative impact of extra labor cost in the EM is not that significant; on the other hand, the wage floor $r_B/\tau$ in the C$^+$M impedes the company's capability of temporal incentive pooling, i.e., the company is no longer able to strategically differentiate the earning rates in different time periods as much as it could in the CM.

To conclude, we have investigated the potential drawback of current regulations. Neither the EM nor the C$^+$M will always enhance full-timers' welfare, let alone other stakeholders' interests can be compromised. Note, though, the uniform classifications essentially require that all workers be

[19] The caveat is, of course, that in the EM, the company can also adjust the service capacities (e.g., hiring extra part-time employees in peak periods) throughout the day. See Appendix OA3.4 for an extension with the dynamic scheduling in the EM.

provided with some employee benefits. The company could have saved the benefits on part-timers and redistributed them to full-timers. We will closely examine this intuition in the next section.

## 5. Implications of Differentiated Schemes

In light of the issues with uniform classifications, in this section, we will study two differentiated schemes that treat full- and part-timers separately. Below we will first analyze an alternative worker classification and then study an operational strategy as a companion.

### 5.1. Hybrid Mode: A Differentiated Worker Classification

In the hybrid mode (HM),[20] we propose that the company treats part-timers as contractors but full-timers as employees. The practical foundation for this scheme is that, e.g., Uber gives workers extra bonuses for their persistent participation (Chen et al. 2020b). Clearly, full-timers are more likely to be rewarded than part-timers. Also note that, we contribute to the work on the hybrid workforce (e.g., Lobel et al. 2022) by (a) capturing the imperfect utilization of employees (i.e., employees also wait for new services) and (b) comparing the HM with the CM, which is essentially a "semi-HM" consisting of contractors (i.e., part-timers) and "pseudo-employees" (i.e., full-timers).

To set up, think of the HM as an upgrade from the EM. For full-timers, the company will hire at least as many of them as it would do in the EM and will pay those who are hired $r_B$ for every time unit on the platform. Recall that $\lambda_E^*$ is the optimal workforce size in the EM. Now the company can also enroll part-timers as contractors, i.e., allow them to self-schedule and compensate them on a per-piece basis. Define $q_{a,t}^H \in [0,1]$ as the participation probability and $\lambda_{a,t}^H = q_{a,t}^H \gamma_{a,t} M_a \tau$ as the effective arrival rate of part-timers in any period $t \in \{\mathsf{h}, \mathsf{l}\}$, respectively. Lemma 1 implies the uniqueness of $q_{a,t}^H$ given any piece-rate wage $w_t$. The company's problem in the HM is thus

$$\max_{\lambda_H \geq \lambda_E^*, w_\mathsf{h}, w_\mathsf{l}} \Pi_H \equiv T \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t((p_t - r_B(\frac{1}{\mu_t - \lambda_H - \lambda_{a,t}^H(w_t)} + \frac{1}{\tau}))\lambda_H + (p_t - w_t)\lambda_{a,t}^H(w_t)). \tag{4}$$

Denote by $\lambda_H^*$ the optimal size of full-time workforce, $S_H^*$ full-timers' welfare, and $\Pi_H^*$ the company's profit in the HM equilibrium. It is useful first to compare the HM with the EM as follows.

Theorem 5 (HM vs. EM). *We have $S_H^* = S_E^*$, $\lambda_H^* \geq \lambda_E^*$ and $\Pi_H^* \geq \Pi_E^*$.*

Theorem 5 conveys a positive message: to classify gig workers discriminatorily will improve over reclassifying all of them as employees because full-timers will earn as much as in the EM while consumers and the company will both be better off.

[20] Despite the same terminology, the "hybrid mode" in Hagiu and Wright (2019) is essentially our C$^+$M where the worker is entitled to a fraction of employee benefits.

The result for full-timers indicates that the company will hire as many full-timers in the HM as in the EM since full-timers' welfare in both scenarios take the form $qM_fB$, where $q$ is the fraction of full-timers hired by the company. The intuition is that $q$ affects the company's profit in two ways: as $q$ increases, on the one hand, it increases the transaction volume throughout the day, but on the other hand, it increases workers' idleness and makes it more costly to hire each full-timer. The fraction $q_E^*$ optimally balances such a trade-off in the EM. In the HM, part-timers' participation increases the period-$t$ average wait time from $1/(\mu_t - q_E^*\lambda_f)$ in the EM to $1/(\mu_t - q_E^*\lambda_f - q_{a,t}^{H,*}\lambda_{a,t})$, and thus makes it more costly to hire $\lambda_E^*$ many full-timers. As such, the company in the HM will lower $q$ to rebalance the trade-off between the transaction volume and the labor cost. Yet with the requirement $\lambda_H \geq \lambda_E^*$, the optimal size of full-time workforce $\lambda_H^*$ in the HM must equal $\lambda_E^*$.

The equivalence of full-timers' welfare implies that the HM inherits the undercutting issue in the EM but curbs the overjoining issue found in the C$^+$M. That is, while full-timers in the HM may still suffer from the company's strict control over their compensation and joining rate, they can be sheltered from their own and their part-time counterpart's overjoining. In particular, in the discussion of Theorem 2(iii), we show that in the C$^+$M the lump-sum benefit $B$ that shall be distributed to full-timers will be cannibalized by part-timers, whereas here in the HM, full-timers' weflare $S_H^* = S_E^*$ and will no longer be adversely affected by part-timers' self-interested participation. Moreover, Theorem 2(i) shows that the C$^+$M can fail to enhance full-timers' welfare with a relatively low $B$ since full- and part-timers are already overjoining in the CM, and Theorem 2(ii) implies that full-timers' welfare may not be improved in the C$^+$M even with a sufficiently large $B$ due to their own overjoining. In stark contrast, Proposition 2(i) and Theorem 5 together imply that full-timers' welfare in the HM can be effectively enhanced as long as $B > 0$.

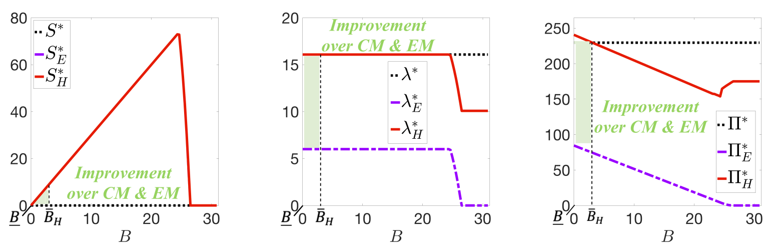
As our previous discussion implies, the caveat of the HM is that the company would hire fewer full-timers without the requirement $\lambda_H \geq \lambda_E^*$, and full-timers' welfare would thus be lower in the HM than in the EM; in other words, the undercutting issue could be intensified. Yet as long it is not too costly for regulators to enforce the requirement $\lambda_H \geq \lambda_E^*$ (e.g., through auditing companies' recruitment), the adverse impact of the HM on full-timers can be limited.

Finally, for consumers and the company, if the company enrolls no part-timers at all, clearly, both the transaction volume and the company's profit in the HM will be the same as in the EM. Given the ability to treat part-timers as contractors, the company will realize a higher transaction volume and make more profit in the HM than in the EM. In fact, both measures in the HM can even be higher than in the CM. The following proposition formalizes this result.

Proposition 3 (HM vs. CM for Consumers and the Company). *Suppose* $q_E^* = 1$ *when* $B = 0$. *There exists a* $\overline{B}_H$ *such that if* $B \le \overline{B}_H$, *we have* $\lambda_H^* \ge \lambda^*$ *and* $\Pi_H^* \ge \Pi^*$.

We explain Proposition 3 as follows: We know that the company will hire as many full-timers in the HM as in the EM (i.e., $\lambda_H^* = \lambda_E^*$). When the lump-sum benefit $B$ is not very high (i.e., $B \le \overline{B}_H$), the optimal size of full-time workforce $\lambda_H^*$ in the HM will be relatively high, but the expected labor cost for each service fulfilled by a full-timer in period $t$, $r_B(W_t + 1/\tau)$, can be lower than the optimal piece-rate wage $w_t^*$ in the CM. More importantly, because full- and part-timers can be compensated differentially, it also becomes relatively less expensive for the company to engage part-timers. As such, the company can either incentivize as many part-timers to participate as in the CM but with a wage lower than $w_t^*$ or strategically enroll more part-timers than in the CM. The company can thus achieve higher transaction volume and make more profit than in the CM.

Corollary 1 (HM vs. CM). *For* $\underline{B}$ *defined in Theorem 1 and* $\overline{B}_H$ *defined in Proposition 3, if* $\underline{B} \le \overline{B}_H$, *we have* $S_H^* \ge S^*$, $\lambda_H^* \ge \lambda^*$ *and* $\Pi_H^* \ge \Pi^*$ *iff* $B \in [\underline{B}, \overline{B}_H]$.




(a) For Full-timers (b) For Consumers (c) For the Company

**Figure 3 Implications of the Hybrid Mode**

*Note.* $\beta_{\mathsf{h}} = 0.1, M_f = 3, M_a = 12, \gamma_{a,\mathsf{h}} = 0.6, \mu_{\mathsf{h}} = 40, \mu_{\mathsf{l}} = 25, \tau = 2, p_{\mathsf{h}} = 45, p_{\mathsf{l}} = 18, r_0 = 12, c_f = 7, c_a = 8$.

Together with Theorem 1, Proposition 3 implies that the HM can be a valid alternative to uniform classifications given that it may enhance full-timers' welfare without making other stakeholders worse off than in the CM. Corollary 1 formalizes this argument, and Fig. 3 helps illustrate it.

### 5.2. Full-Timers' Priority: A Differentiated Operational Policy

As a companion to the classification approaches, we now study an *operational* scheme in the CM: when assigning requests, the company prioritizes full-timers, who depend on gigs for a living, over part-timers, who do gigs only for supplemental income.[21] Denote this scenario by C$^\pi$M, with "$\pi$" referring to the priority. Note that in practice, platforms are experimenting with similar approaches; see Krishnan et al. (2022) for an example on Lyft. We complement Krishnan et al. (2022) by differentiating between full- and part-timers and focusing on the welfare implications for full-timers.

[21] One may be concerned that such a dispatch policy renders the platform too much control over workers. We can let the platform randomly prioritize a subgroup of full-timers over the other full-timers and all the part-timers.

We first analyze how workers' joining decisions will be affected. For full-timers, because of their priority, the virtual queue for requests now operates independently of part-timers' participation. In particular, the average wait time for a request in period $t$ becomes $W^{\pi}_{f,t} = 1/(\mu_t - q_\pi \cdot q^{\pi}_{f,t}\lambda_f)$ for any commitment probability $q_\pi$ and participation probability $q^{\pi}_{f,t}$ that meet $q_\pi \cdot q^{\pi}_{f,t} < \mu_t/\lambda_f$ and infinity otherwise. For part-timers, their expected wait times now become longer as full-timers are being prioritized. To be specific, the average wait time for any part-timer in period $t$ is $W^{\pi}_{a,t} = 1/((\mu_t - q_\pi \cdot q^{\pi}_{f,t}\lambda_f)(\mu_t - q_\pi \cdot q^{\pi}_{f,t}\lambda_f - q^{\pi}_{a,t}\lambda_{a,t})/\mu_t)$, for any participation probability $q^{\pi}_{a,t} < (\mu_t - q_\pi \cdot q^{\pi}_{f,t}\lambda_f)/\lambda_{a,t}$ and infinity otherwise; see section 4.4.3 in Gross and Harris (1985) for details.

One may already notice that this operational scheme can simultaneously counteract both undercutting and overjoining. Unlike in the EM, full-timers can self-schedule and will be paid on a piece-rate basis; thanks to part-timers' participation, again, there can be high earning rates that result in a positive surplus. And in contrast to the C$^+$M, full-timers' utilization rate and hence the expected utility on the platform will no longer be adversely affected by part-timers' overjoining (though full-timers themselves may still overjoin). Denote by $S^*_\pi$ full-timers' welfare in the C$^\pi$M equilibrium. We formalize the potential of C$^\pi$M to counteract overjoining as follows.

Proposition 4 (C$^\pi$M vs. CM for full-timers). *If part-timers' participation probability $q^{\pi,*}_{a,t} > 0$ in the C$^\pi$M equilibrium for both $t \in \{\mathsf{h}, \mathsf{l}\}$, we have $S^*_\pi \geq S^*$; in particular, if $S^* > 0$, we have $S^*_\pi > S^*$ and that $(S^*_\pi - S^*)/S^*$ will increase in part-timers' labor pool size $M_a$.*

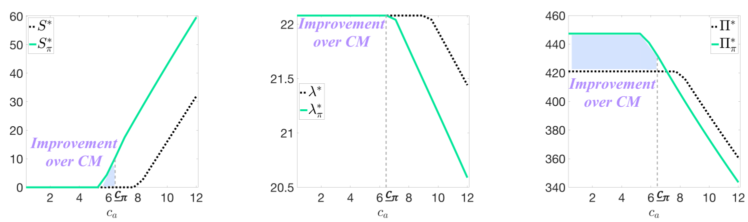
Proposition 4 says that in the C$^\pi$M, as long as there are some part-timers participating throughout the day (i.e., $q^{\pi,*}_{a,t} > 0$ for both $t \in \{\mathsf{h}, \mathsf{l}\}$), full-timers' total welfare will be higher than in the CM. In particular, as per Proposition 4, conditional on that full-timers' welfare is positive in the CM (i.e., $S^* > 0$), their welfare will increase at a larger scale in the C$^\pi$M as more people become available part-time (i.e., $M_a$ increases). Such results are in stark contrast to Theorem 2, which implies that full-timers' welfare either may not be enhanced or can be enhanced but at a slower rate due to part-timers' overjoining. As such, one can see that the dispatch priority is enhancing the positive externality part-timers impose on full-timers, which we highlighted in Proposition 1(i).

Note, though, if due to full-timers' priority part-timers' waiting disutility increases too much (happens when, e.g., full-timers' labor pool $M_f$ is rather large), the company may only enroll few of them if at all (i.e., $q^{\pi,*}_{a,t}$ close to 0), given the higher financial incentive they now require. Yet in such cases the wages will generally be too low to make full-timers better off than in the CM.

Finally, we briefly discuss the implications for consumers and the company.

Proposition 5 (C$^\pi$M vs. CM for Consumers and the Company). *There exists $\underline{c}_\pi$ such that if $c_a \leq \underline{c}_\pi$, we have $\lambda^*_\pi \geq \lambda^*$ and $\Pi^*_\pi \geq \Pi^*$.*

Proposition 5 says if part-timers' opportunity cost is relatively low (i.e., $c_a \leq \underline{c}_\pi$), the C$^\pi$M can make both consumers and the company better off than in the CM. To intuit, suppose the company tends to enroll at least as many workers, full- or part-time, in the C$^\pi$M as in the CM to serve the market. Because of their priority, full-timers' average wait time is shorter in the C$^\pi$M than in the CM, and the company can use lower piece-rate wages in the C$^\pi$M to make them commit and participate. In contrast, part-timers' average wait times become longer; yet thanks to a sufficiently low opportunity cost $c_a$, the total labor cost to incentivize part-timers to participate up to similar levels as in the CM shall not increase too much. As such, the company shall be able to maintain the transaction volume at a lower cost and thus make more profit in the C$^+$M than in the CM, as the extra expense of incentivizing part-timers can be less than the savings from enrolling full-timers.




(a) For Full-timers (b) For Consumers (c) For the Company

**Figure 4 Implications of the Priority Mode**

*Note.* $\beta_{\mathsf{h}} = 0.2, M_f = 4, M_a = 16, \gamma_{a,\mathsf{h}} = 0.6, \mu_{\mathsf{h}} = 40, \mu_{\mathsf{l}} = 25, \tau = 2, p_{\mathsf{h}} = 45, p_{\mathsf{l}} = 18, r_0 = 8, c_f = 6.$

We use the numerical example in Fig. 4 to corroborate results in Proposition 5. Also note that, in this example, if $c_a \leq \underline{c}_\pi$, full-timers' welfare will also be higher in the C$^\pi$M than in the CM, implying that the C$^\pi$M can be a valid alternative to uniform classifications as well.

## 6. Numerical Study

In this section, we calibrate the parameters in our model using real-world data and numerically review our insights. Our motivation is the California Assembly Bill No. 5. The bill was signed into law in 2019, attempting to reclassify gig workers as employees; however, Uber and other gig companies successfully sought exemption from the law in 2020. Our experiment is conducted at the city level with data for 2019. We predict market outcomes for 137 California cities in which Uber claims to be available[22] and our model predicts active operations. Table 2 provides the summary statistics at the city level. See Online Appendix OA1 for the parameter calibration and data sources.

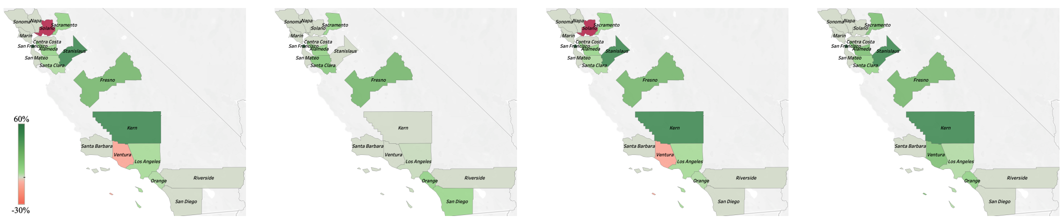
Figures 5, 6 and 7 display our model's predictions for full-timers, consumers, and the company, respectively. In each figure, sub-figures (a) and (b) show the implications of uniform classifications,

[22] https://www.uber.com/global/en/cities/.

| | Mean | SD | Median | Min | Max |
|---|---|---|---|---|---|
| $\beta_{\mathsf{h}}$ | 0.200 | 0 | 0.200 | 0.200 | 0.200 |
| $\gamma_{a,\mathsf{h}}$ | 0.600 | 0 | 0.600 | 0.600 | 0.600 |
| $M_f$ | 0.289 | 0.765 | 0.148 | 0.051 | 8.591 |
| $M_a$ | 3.223 | 7.179 | 1.751 | 0.821 | 76.467 |
| Event rate parameters (#/hour) | | | | | |
| $\mu_{\mathsf{h}}$ | 3.589 | 8.110 | 1.916 | 0.928 | 86.835 |
| $\mu_{\mathsf{l}}$ | 2.567 | 5.801 | 1.370 | 0.664 | 62.111 |
| $\tau$ | 2.891 | 0.226 | 2.932 | 2.092 | 3.457 |
| Pecuniary parameters (2019 USD) | | | | | |
| $p_{\mathsf{h}}$ | 24.559 | 1.826 | 24.555 | 9.522 | 30.692 |
| $p_{\mathsf{l}}$ | 9.446 | 0.702 | 9.444 | 3.662 | 11.805 |
| $r_0$ | 12.139 | 0.586 | 12.000 | 12.000 | 15.000 |
| $c_f$ | 9.711 | 0.469 | 9.600 | 9.600 | 12.000 |
| $c_a$ | 23.234 | 4.902 | 23.065 | 14.733 | 39.498 |
| $B$ | 6.961 | 0.117 | 7.005 | 6.820 | 7.558 |

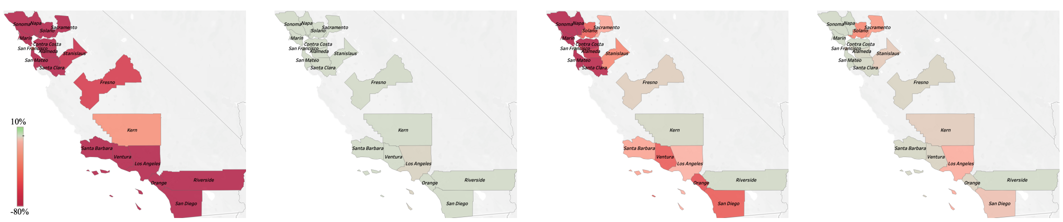
*Note.* $N = 137$ cities.

**Table 2 Summary Statistics of Calibrated Parameters**



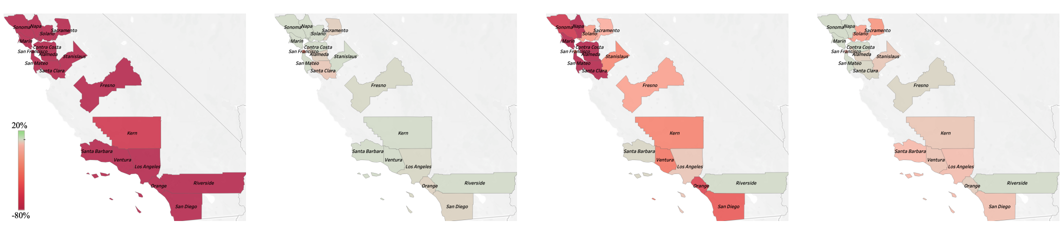

(a) (EM − CM)/CM (b) (C$^+$M − CM)/CM (c) (HM − CM)/CM (d) (C$^\pi$M − CM)/CM

**Figure 5 Implications for Full-timers**




(a) (EM − CM)/CM (b) (C$^+$M − CM)/CM (c) (HM − CM)/CM (d) (C$^\pi$M − CM)/CM

**Figure 6 Implications for Consumers**




(a) (EM − CM)/CM (b) (C$^+$M − CM)/CM (c) (HM − CM)/CM (d) (C$^\pi$M − CM)/CM

**Figure 7 Implications for the Company**

and sub-figures (c) and (d) demonstrate those of differentiated schemes. We quantify the implications as the percentage change in certain variables relative to the benchmark of the CM. The results are aggregated at the county level by averaging city-level outcomes. In the online version, we color an area green if a change is positive, red if it is negative, and grey if the change is not very significant in scale. According to sub-figures (a) in these three figures, we see that the EM may backfire and hurt full-timers (e.g., 10.3% welfare decrease in the county of Ventura), and the negative impacts on consumers and the company will be substantial. Sub-figures(b) imply that the wage floor in the C$^+$M may not enforce in many cities, and correspondingly the negative impacts on other stakeholders are less significant than in the EM. Indeed, our model predicts that the wage floor $r_B/\tau$ will enforce only in 16.67% of all the cities during off-peak periods. Then sub-figures (c) show that full-timers will earn as much as in the EM and, in a majority of cities, will be better off than in the CM or C$^+$M (i.e., the *overjoining* issue can be curbed). The negative impacts on other stakeholders also become less pronounced than in the EM: relative to the EM, the negative impacts on consumers and the company will, on average, contract by 58.55% and by 60.38%, respectively. Finally, sub-figures (d) show that, unlike the HM, the C$^\pi$M can avoid backfiring and hurting full-timers, and its negative impacts on consumers and the company are even more modest.

Note that, for this empirical analysis, we essentially abstract each city as a "representative block" and predict outcomes therein to illustrate the first-order insights our model has offered. For more accurate evaluations, one may wish to replicate our model and fine-tune the parameters to match the primitives in different blocks and, importantly, add the dynamics of workers' relocation (e.g., Bimpikis et al. 2019). Results then depend on the market demand for trips with different origins and destinations, workers' entry and relocations, the company's dispatching policy and so forth.

## 7. Discussions

In this section, we will first study the part-timers' welfare and the aggregate worker welfare and then relax certain assumptions in the baseline to examine the robustness of our main findings.

### 7.1. Implications for Part-timers & Aggregate Worker Welfare

Proposition 6 (Implications for Part-timers). *Recall that $\tilde{S}^*$, $\tilde{S}^*_E$, $\tilde{S}^*_+$, $\tilde{S}^*_H$ and $\tilde{S}^*_\pi$ denote part-timers' welfare in the CM, EM, C$^+$M, HM and C$^\pi$M, respectively. We have (i) If $c_a \geq \bar{\bar{c}}$ for some $\bar{\bar{c}}$, $\tilde{S}^* = 0$; otherwise, there exists $\overline{\tilde{M}}$ such that if part-timers' labor pool size $M_a \leq \overline{\tilde{M}}$, we have $\tilde{S}^* > 0$. (ii) $\tilde{S}^*_E = \tilde{S}^*_H = 0$. (iii) For $\underline{B}_+, \overline{B}_+$ defined in Theorem 2, if $B \geq \overline{B}_+$, we have $\tilde{S}^*_+ \leq \tilde{S}^*$; for $B \leq \underline{B}_+$, we have $\tilde{S}^*_+ \geq \tilde{S}^*$ and for $\tilde{S}^* > 0$, $(\tilde{S}^*_+ - \tilde{S}^*)/\tilde{S}^*$ decreases in part-timers' labor pool size $M_a$. (iv) There exists $\bar{\bar{c}}_\pi \leq \bar{\bar{c}}$ such that if $c_a \in [\bar{\bar{c}}_\pi, \bar{\bar{c}}]$, we have $\tilde{S}^*_\pi \leq \tilde{S}^*$.*

It is worthwhile to highlight Proposition 6(iii). In Theorem 2(iii) we show that the positive impact of C$^+$M on full-timers will taper off as more people become available on a part-time basis;

here Proposition 6(iii) implies that the positive impact of C$^+$M on part-timers themselves also attenuates as their population grows.

Combining Proposition 6 and our previous results on full-timers' welfare, we summarize the findings on workers' aggregate welfare as below.

Proposition 7 (Implications for Aggregate Worker Welfare). *Let* $\mathtt{S}^*$, $\mathtt{S}_E^*$, $\mathtt{S}_+^*$, $\mathtt{S}_H^*$ *and* $\mathtt{S}_\pi^*$ *denote workers' total welfare in the CM, EM, C*$^+$*M, HM and C*$^\pi$*M, respectively. We have (i)* $\mathtt{S}_E^* = \mathtt{S}_H^*$ *and there exist* $\underline{\mathtt{B}} \le \overline{\mathtt{B}}$ *such that* $\mathtt{S}_E^* = \mathtt{S}_H^* \ge \mathtt{S}^*$ *if and only if* $B \in [\underline{\mathtt{B}}, \overline{\mathtt{B}}]$. *(ii) For* $\underline{B}_+, \overline{B}_+$ *defined in Theorem 2, if* $B \ge \overline{B}_+$, *we have* $\mathtt{S}_+^* \le \mathtt{S}^*$; *for* $B \le \underline{B}_+$, *we have* $\mathtt{S}_+^* \ge \mathtt{S}^*$ *and for* $\mathtt{S}^* > 0$, *if* $M_f = \gamma M$ *for some constant* $\gamma \in (0,1)$, $(\mathtt{S}_+^* - \mathtt{S}^*)/\mathtt{S}^*$ *decreases in the aggregate labor pool size* $M$. *(iii) There exist* $\underline{c}'_\pi$ *and* $\overline{c}'_\pi$ *such that if* $c_a \in (\underline{c}'_\pi, \overline{c}'_\pi)$, *we have* $\mathtt{S}_\pi^* > \mathtt{S}^*$.

We would like to comment on Proposition 7(iii) in particular, which says the aggregate workers' welfare would be higher in the C$^\pi$M than in the CM if part-timers' opportunity cost is at a moderate level (i.e., $c_a \in [\underline{c}'_\pi, \overline{c}'_\pi]$). To intuit, since part-timers' opportunity cost is not very high, the company will afford to enroll a number of them in the C$^\pi$M; this implies that part-timers' earning rates cannot be very low, even though they are deprioritized and their utilization rates become lower in general. In the meanwhile, full-timers' welfare is clearly enhanced thanks to their priority over part-timers and the higher piece-rate wages the company pays in order to incentivize part-timers.

### 7.2. Robustness Checks

We now conduct robustness checks of our main findings, i.e., the issues with uniform regulations (Theorems 1 & 2) and the performances of differentiated schemes (Section 5), in several extensions.

**7.2.1. Endogenous Service Price** We first endogenize service prices $p_{\mathsf{h}}$ and $p_{\mathsf{l}}$ charged from consumers. To this end, we denote by $\mu_{t,0}$ the *market potential* in period $t \in \{\mathsf{h}, \mathsf{l}\}$. Upon arriving at the platform, each consumer draws a valuation $V$ randomly from a distribution $F$ and will patronize if and only if (A) there are idled workers who are ready to begin services immediately and (B) $V$ exceeds the service price $p_t$, and will leave the platform otherwise.[23] As such, the effective demand rate in period $t$ is $\mu_t(p_t) = \mu_{t,0}\overline{F}(p_t)$. We then plug such effective demand rates into the company's profit functions and let the company jointly optimize prices and wages in different schemes. Numerical results in Appendix OA3.1 confirm most of our main insights.

It is worthwhile to highlight one new finding: contrary to Theorem 2, the C$^+$M may now hurt full-timers even for an intermediate benefit level $B$; see Fig. OA.3(b) for an illustration. To understand

[23] This modelling approach aligns with the gig economy literature (see, e.g., Cachon et al. 2017, Banerjee and Johari 2019).

this, we have discussed in the baseline that workers are *overjoining* in the C$^+$M, and this can reduce workers' utilization rates, i.e., each of them spends more time waiting to be dispatched. Now, the company also raises service prices as the wage floor $r_B/\tau$ pushes up the piece-rate wages. This leads to decreases in the market demand $\mu_t(p_t)$, which further reduce workers' utilization rates. Eventually, the decrease in utilization rates will become more significant than the increase in piece-rate wages, and this results in a decrease in workers' earnings and their welfare.

**7.2.2. Exact Closed Queueing Network Analysis** In Section 3, we clarify that our model is essentially a finite-source queue; we follow the literature and approximate the system as an infinite-source queue to facilitate the analysis in the baseline. Here we will execute the exact closed queueing network analysis to stress-test the managerial insights we have derived. Suppose in period $t$ we have $M_t$ workers (full-time and part-time combined) participating on the platform. According to Gross and Harris (1985), when workers are dispatched FCFS, the average wait time for any worker to be dispatched is $W_t = L_t/(\tau(M_t - L_t)$, where $L_t = \pi_0 \sum_{i=1}^{M_t} i \cdot \binom{M_t}{i} i!(\tau/\mu)^i$ denotes the expected number of workers waiting to be matched and $\pi_0 = 1/(1 + \sum_{i=1}^{M_t} \binom{M_t}{i} i!(\tau/\mu)^i)$ denotes the stationary probability of there being no worker waiting, or equivalently, all $M_t$ workers being fulfilling services; see Appendix OA3.2 for more details. We observe numerically all of our key insights continue to hold given this exact formulation.

**7.2.3. Alternative Dispatching Policy** Another underlying assumption for our model is that requests are assigned to workers on a first-come-first-served (FCFS) basis. In the ride-hailing setting, for example, this implies that new requests in a zone will be assigned to drivers who have arrived earlier than others; in reality, however, it is oftentimes those workers who are geographically closer to the riders that get dispatched first. As such, here we will replace the FCFS queueing-foundational wait time $W_t$ in our model with that $\tilde{W}_t$ under an alternative dispatching policy. In particular, we consider the *Nearest Neighbor* matching in ridehailing (e.g., Feng et al. 2021, Besbes et al. 2022a) and approximate the wait time in a fluid manner as $\tilde{W}_t = 1/\mu_t + (1/\tilde{L}_t) \cdot (\bar{e}/\sqrt{\tilde{L}_t}) + (1 - 1/\tilde{L}_t) \cdot \tilde{W}_t$, where $\tilde{L}_t$ denotes the average number of workers waiting to be dispatched; by Little's Law, we have $\tilde{L}_t = \lambda_t \tilde{W}_t$. The first term on the right-hand side of the formulae is the expected time for the next consumer to pop up, the second term measures the average *en route* time to pick up the consumer if a worker is the nearest (with $\bar{e}$ being the constant for the pick-up time when $\tilde{L}_t = 1$), and the third term renews the waiting process if the worker is not the nearest. Again, our numerical studies confirm the robustness of all key insights. See Appendix OA3.3 for more details.

## 8. Concluding Remarks

This paper speaks to the recent policy debate over worker classification in the on-demand economy. We highlight the difference between full-timers and part-timers and focus on the welfare of the latter, who have served effectively as employees yet have not been provided with the benefits that they need and deserve. We show that classification schemes overlooking the heterogeneity among gig workers will cause problems both with workers being undercut and with workers overjoining. We propose schemes that differentiate both in classifications and from an operational perspective and demonstrate the potential of these schemes to improve over uniform classifications.

Our theoretical framework serves as a basis for better understanding the worker classification issue. For ease of exposition, we assume that part-timers will be absent in the EM because of their strong preference for the flexibility of being contractors. However, it would be useful to further microfound part-timers' decisions and support this assumption analytically. Another interesting extension would be to consider generally distributed opportunity costs $C_f$ and $C_a$. Compared to the baseline with degenerate cost distributions, it will be more challenging for the company to control workers' participation in the CM indirectly via piece-rate wages. The company may thus make a higher profit in the EM than in the CM, given its ability to directly administer the workforce.

Many other directions are worthy of further exploration. One promising avenue is to consider the setting with competing platforms. In fact, the on-demand economy is marked by the coexistence of competing platforms and workers having the flexibility to switch between platforms (i.e., multihoming). In the CM, the platform competition (particularly during peak periods) can result in higher earning rates than in the single platform setting with temporal incentive pooling and thus leave full-timers a positive surplus. The overjoining issue in the C$^+$M can be moderated as multihoming workers will split among different platforms. In the EM, companies may prohibit workers' multihoming so as to avoid paying "idle" workers who are actually busy serving on competing platforms, yet they may also compete upfront to hire full-timers as employees, which may alleviate the undercutting issue. Finally, we expect that treating workers differentially will still be valuable, and it will be interesting to study the competition among platforms with hybrid workforces.

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# Online Appendix to

**"Implications of Worker Classification in On-Demand Economy"**

## OA1. Literature Review: Worker Classification & The Theory of Firm

Predating the intense debate on worker classification is the puzzle of organizational design. While the Agency Theory (Alchian and Demsetz 1972) and the long-run relational Theory of Firm (ToF hereafter; Williamson 1975, Holmstrom and Tirole 1989) would predict that the contractor (or *owner-operator*) mode shall be the dominant organization form for transportation service firms, in reality what really prevails is a hybrid mode of contractors and a significant number of employee drivers (see, e.g., Sherer et al. 1998, Nickerson and Silverman 2003). Over the years, organizational economists have investigated why firms would deploy different classes of human resources. Consistent with classic organizational theories, researchers find that contractors generally have stronger incentives to perform quality jobs (given that they are *residual claimants*) and they also help firms save payroll taxes, extra administrative costs and so forth (see, e.g., Baker and Hubbard 2004, Cappelli and Keller 2013, Rawley and Simcoe 2013). On the other hand, though, hiring employees gives firms considerable mileage to better coordinate the workforce, improve service qualities, cope with environmental uncertainties and build the firm reputation (see, e.g., Sherer et al. 1998, Nickerson and Silverman 2003), which create and buttress firms' competitive advantages (Barney 1991). Such discussions are reminiscent of the classic "Make-or-Buy" problem and the theory on Vertical Integration (Lafontaine and Slade 2007): indeed, for firms hiring employees is essentially to make products/services in-house (i.e., integration), while deploying contractors is analogous to procuring from outside suppliers for product/services (i.e., dis-integration).

Research along this line implicitly presumes that different human resources have been treated and classified in a legally proper manner in the first place, be them as employees or as contractors. The acute issue today is, however, that platform companies' practices of worker classification are legally very controversial, and so far there lack well-functioning laws in general to discipline companies' behaviors in the new terrain of on-demand economy.

The crux here is, while the ToF has offered profound insights for understanding the scale and the form of an organization (Coase 1937, Williamson 1975), it offers much less clearcut prescriptions on how to (legally) define the statuses of different human resources deployed in the organization. For one thing, the classic ToF delineates the boundary of a firm by dichotomizing objects related to the firm's operations into two batches, with and without the firm's direct control; in the on-demand economy, however, gig workers are simultaneously controlled by platforms over some aspects of their jobs and have much flexibility in the other aspects (mainly the work schedule). Perhaps more importantly, even with gig workers being included in the boundary of firms like Uber, the ToF does not directly inform what object within a firm's boundary really defines its commercial nature. In fact, Uber and other companies insist that they are not transportation service companies, but are instead intermediaries whose goals are to better match buyers (i.e., consumers) and sellers (i.e., workers) and support individual entrepreneurships by providing independent workers with an unprecedented amount of flexibility over their work schedules. See Tomassetti (2016) for how Uber's narrative has impeded the legal process of worker reclassification.

Though this paper does not intend to investigate how the classic ToF might resolve its potential limitations so as to advise the worker classification issue, we contend that broader impacts of worker classification shall be examined from many other social-economic perspectives to complement the insights from the ToF. We wish to demonstrate through our work that the worker heterogeneity can be one such perspective going forward, and to this end we shed light on potential consequences of different classification schemes, with a main focus on full-timers' welfare.

# OA2. Details on Section 6 Numerical Study

## OA2.1 Methodologies for Parameter Calibration

We now discuss how we calibrate the parameters for the numerical study in Section 6.

- The fraction of a day in peak times $\beta_{\mathsf{h}}$: we set $\beta_{\mathsf{h}} = 0.2$. Peak times normally last for 2 to 3 hours in the morning (e.g., 7 to 9 or 10AM) and then at dawn (e.g., 5PM to 7 or 8PM); see, e.g., Rayle et al. (2016), Allon et al. (2023). We set total peak hours being 4.8 hours in a day and round up $\beta_{\mathsf{h}}$ to be 0.2. We find that the numerical outcomes are not sensitive to the specific value of $\beta_{\mathsf{h}}$.
- The fraction part-timers being available in peak periods $\gamma_{a,\mathsf{h}}$: According to Chen et al. (2020a), part-timers ("infrequent drivers" as they refer to) are more likely to come to work during 8-10AM in the morning and 6-8PM in the evening, which are typical peak times in a city. In light of stylized fact, we set $\gamma_{a,\mathsf{h}} = 0.6$. Robustness checks with other $\gamma_{a,\mathsf{h}}$ ranging from 0.6 to 0.8 show that the numerical outcomes are not very sensitive to the exact value of $\gamma_{a,\mathsf{h}}$ either.
- Full- and part-timers' labor pool sizes $M_f$ and $M_a$ and the request rates in peak and off-peak periods $\mu_{\mathsf{h}}$ and $\mu_{\mathsf{l}}$. We calibrate these parameters first for San Francisco and then for other cities.

For rates of service requests, The San Francisco city provides ridehailing data on hourly pickups and drop-offs for every "transportation analysis zone" (TAZ) over a week. As we have commented at the end of Section 6, in this paper we have essentially abstracted each city as a "representative block". As such, we calibrate $\mu$ using the average transaction volume among all TAZs. In particular, $\mu_{\mathsf{h}}$ uses the transaction volume from 7-9AM in the morning and 5-8PM at dawn, and $\mu_{\mathsf{l}}$ uses the data during the rest of time in a day. Given that transactions are more likely to take place downtown than in other areas, we weigh each TAZ with respect to the intensity of economic activities, which we proxy directly using the transaction volume. Then as in Hu et al. (2021), we estimate $\mu_{\mathsf{h}}$ and $\mu_{\mathsf{l}}$ as the weighted average number of pickups.

For labor pool sizes, we similarly estimate $M_f$ and $M_a$ at the block-level. To this end, we first collect the total number of registered drivers in San Francisco $\mathbf{M}$, and then average it over TAZs to obtain the block-level total labor pool size $M$. We assume full-timers' fraction in the labor pool is consistent with $\gamma$, the percentage of persons without health insurance under the age of 65. We then obtain $M_f = \gamma M$ and $M_a = (1-\gamma)M$.

To further clarify a few items: first, since the data sources above document the aggregate transaction volume of Uber and Lyft, we rescale the pickups and drop-offs by Uber's market share in the U.S., which is about 69.9% in 2019[24]. Second, the data provided by San Francisco was collected in 2016, and we re-scale all parameters according to the ratio of San Francisco's population in 2019 to

[24] https://www.statista.com/statistics/910704/market-share-of-rideshare-companies-united-states/.

that in 2016. Finally, for the other cities in California, we estimate these parameters by re-scaling according to the ratio of the city's population to that of San Francisco in 2019.

• Next, the service rate $\tau$ and service prices $p_{\mathsf{h}}$ and $p_{\mathsf{l}}$. Rayle et al. (2016) estimate that the average length of a ride-hailing trip $1/\tau_{SF}$ in San Francisco is about 22.1 minutes. For any other *city* in California, we estimate $\tau_{city}$ using the formula

$$1/\tau_{city} = (Congestion_{city}/Congestion_{SF}) \cdot (AvgTravelDist_{city}/AvgTravelDist_{SF}) \cdot (1/\tau_{SF}).$$

Here $Congestion_{city}$ is the congestion index of a particular city, which negatively correlates the the average travelling speed, and $AvgTravelDist_{city}$ is the average daily travel distance among citizens in that city. We collect both the congestion data and the travel distance data from the US Bureau of Transportation Statistics.

For service prices, we first estimate an average price $p$ in each city. Uber's service price (i.e., trip fare) $p$ consists of two components: a fixed and a variable fee. The fixed fee is the sum of a base fare and a booking fee for each trip. We assume the fixed fee is constant across cities as this component mostly reflects the cost of platform management. The variable fee depends on the average trip length and can thus be estimated in a fashion similar to $\tau$. Due to the availability of data, we take the average trip fare in the city of Los Angeles as a benchmark. The exact formula for calibration is $p_{city} = FixFee + (AvgTravelDist_{city}/AvgTravelDist_{LA}) \cdot VarFee_{LA}$. We resale the price for each city according to the consumer price indices in 2019.

We then calibrate the peak and off-peak period prices $p_{\mathsf{h}}$ and $p_{\mathsf{l}}$ as follows. We suppose the transaction volume in peak periods account for $\theta_{\mathsf{h}} = 80\%$ of the total volume in a day, and that in off-peak periods is naturally $1 - \theta_{\mathsf{h}} = 20\%$. We further assume that the average surge multiplier is $k = 2.6$ , i.e., $p_{\mathsf{h}} = k \cdot p_{\mathsf{l}}$. Then given that $\theta_{\mathsf{h}} p_{\mathsf{h}} + (1-\theta_{\mathsf{h}}) p_{\mathsf{l}} = p$, we compute $p_{\mathsf{l}} = p/(k \cdot \theta_{\mathsf{h}} + (1-\theta_{\mathsf{h}}))$.

To check whether the calibration for service prices is sensible, we run the following regression: $p_i = \eta_0 + \eta_1 \hat{p}_i + \epsilon_i$ for $i = 1, ..., 137$, where $p_i$ is the calibrated average price for city $i$, and $\hat{p}_i = \hat{\theta}_{\mathsf{h},i} p_{\mathsf{h},i} + (1 - \hat{\theta}_{\mathsf{l},i}) p_{\mathsf{l},i}$ and $\hat{\theta}_{\mathsf{h},i}$ are the predicted average price and the predicted peak volume portion based on the calibrated parameters, respectively. The regression results in a statistically significant (p-value $< 0.001$) coefficient $\hat{\eta}_1 = 1.001$ and also an insignificant (p-value $= 1$) intercept $\hat{\eta}_0 = -1e-13$. This implies that the calibrated prices ($p_i$) and the predicted prices ($\hat{p}_i$) match with each other, which suggests that our initial estimation for the peak volume portion $\theta_{\mathsf{h}}$ and the average surge multiplier $k$ generally make sense.

• We estimate full-timers' earning rate for the labor market outside opion $r_0$ as the social minimum wage set in each city. For full-timers' opportunity costs $c_f$, according to Chen et al.

(2019)'s estimation, following a rigid work schedule can downsize workers' surplus (i.e., $r_0 - c_f$ for full-timers) to around 20% of their total earnings (i.e., $r_0$ for full-timers). We thus calibrate $c_f$ as $c_f = (1-20\%)\cdot r_0$. For part-timers' opportunity cost $c_a$, we calibrate it as the median-level hourly wage: for each *city*, we first collect the median household income $M_{city}$, and then estimate $c_a$ as the individual hourly wage $m_{city}$ using the formula $m_{city} = M_{city}/(1916 * Avg\ Household\ Labor_{city})$, where 1916 is the average annual hours worked according to the U.S. Bureau of Labor Statistics and $Avg\ Household\ Labor_{city}$ estimates the average number of people in a household who are active in the labor market.

According to our calibration, the average ratio of part-timers' opportunity cost to full-timers' opportunity cost, $c_a/c_f$, is about 2.2. This aligns with the empirical finding in Chen et al. (2020a) that *"frequent drivers* [(i.e., full-timers)] *have half as large reservation wages as compared to the infrequent drivers* [(i.e., part-timers)]*."*

- Finally, for the lump-sum benefit $B$, we first compute the difference between the hourly lump-sum employee benefit in "production, transportation, and material moving" industries and the minimum employee benefit available in the labor market across American (recall that we have normalized the employee benefit from full-timers' outside job $b = 0$). We then rescale the parameter for each city according to the consumer price indices in 2019.

## OA2.2 Data Sources for Parameter Calibration

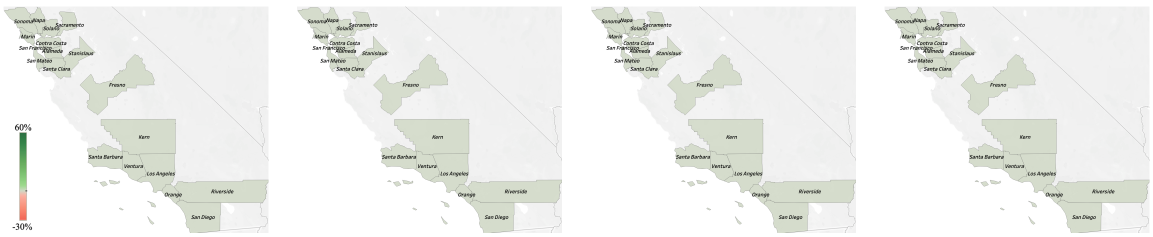
| Parameter | Source |
|---|---|
| $\beta_{\mathsf{h}}$ | Rayle et al. (2016), Allon et al. (2023) |
| $\gamma_{a,\mathsf{h}}$ | Chen et al. (2020a) |
| $M_f$ & $M_a$ | # Registered drivers: https://www.sfcta.org/sites/default/files/2019-02/TNCs_Today_112917_0.pdf |
| | % Without health insurance under 65: https://www.census.gov/quickfacts/fact/table/CA/HEA775220 |
| | % Low wage: https://laborcenter.berkeley.edu/low-wage-work-in-california-data-explorer/ |
| $\mu_{\mathsf{h}}$ & $\mu_{\mathsf{l}}$ | Ridehailing: https://www.sfcta.org/sites/default/files/2019-06/trip_stats_taz_0.csv, Population by city |
| $\tau$ | Mean trip duration: Rayle et al. (2016), |
| | Congestion index: https://www.bts.gov/content/annual-roadway-congestion-index, Average travel distance |
| $p_{\mathsf{h}}$ & $p_{\mathsf{l}}$ | Trip fare: https://www.ridesharingdriver.com/how-much-does-uber-cost-uber-fare-estimator/ |
| | Average travel distance, Consumer price index |
| $r_0$ & $c_f$ | Chen et al. (2019), State min wage: https://www.dir.ca.gov/dlse/faq_minimumwage.htm, |
| | Local min wage: https://laborcenter.berkeley.edu/inventory-of-us-city-and-county-minimum-wage-ordinances/ |
| $c_a$ | Median household income: https://www.census.gov/quickfacts/fact/table/CA,US/INC110220 |
| | Annual hours worked: https://www.bls.gov/opub/mlr/cwc/work-schedules-in-the-national-compensation-survey.pdf |
| | Persons per household: https://www.census.gov/quickfacts/fact/table/CA/HSD310220 |
| | % Civilian labor force: https://www.census.gov/quickfacts/fact/table/CA/LFE041220, Chen et al. (2020a) |
| $B$ | https://www.bls.gov/web/ecec/ececqrtn.pdf, Consumer price index |

Population: https://www2.census.gov/programs-surveys/popest/tables/2010-2019/cities/totals/SUB-IP-EST2019-ANNRES-06.xlsx
Average travel distance: https://data.bts.gov/Research-and-Statistics/Trips-by-Distance/w96p-f2qv
Consumer price index: https://www.dir.ca.gov/oprl/capriceindex.htm

**Table 3 Data Sources for Parameter Calibration**

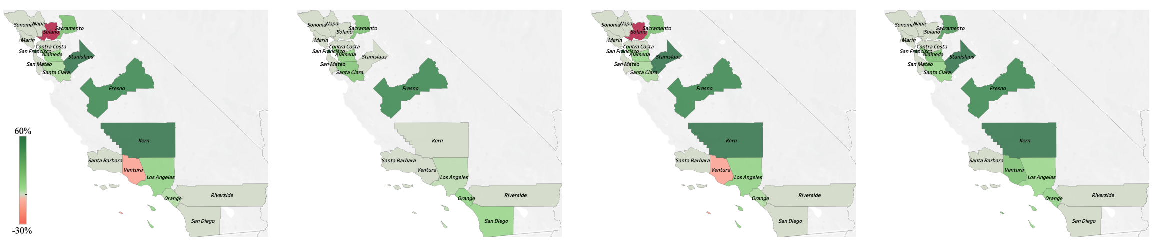
## OA2.3 Results for Part-timers & Aggregate Worker Welfare

Here we supplement with the results for part-timers as well as for the aggregate worker welfare. Fig. OA.1 shows that in general the impacts of different classification/operational schemes on part-timers are not very significant. This is mainly because part-timers' opportunity cost $c_a$ is fairly high according to our calibration and therefore they will mostly end up with trivial surpluses in any scenario. As such, the implications for the aggregate worker welfare (Fig. OA.2) generally align with what are for the full-timers (Fig. 5).




(a) (EM − CM)/CM (b) ($C^{+}$M − CM)/CM (c) (HM − CM)/CM (d) ($C^{\pi}$M − CM)/CM

**Figure OA.1 Implications for Part-timers**



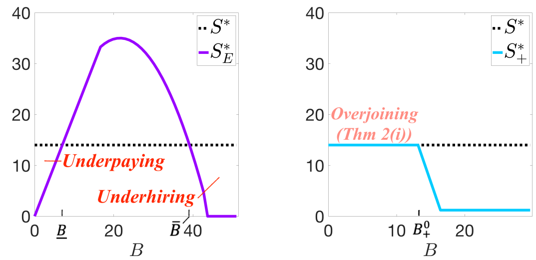

(a) (EM − CM)/CM (b) ($C^{+}$M − CM)/CM (c) (HM − CM)/CM (d) ($C^{\pi}$M − CM)/CM

**Figure OA.2 Implications for Aggregate Worker Welfare**

# OA3. Robustness Checks

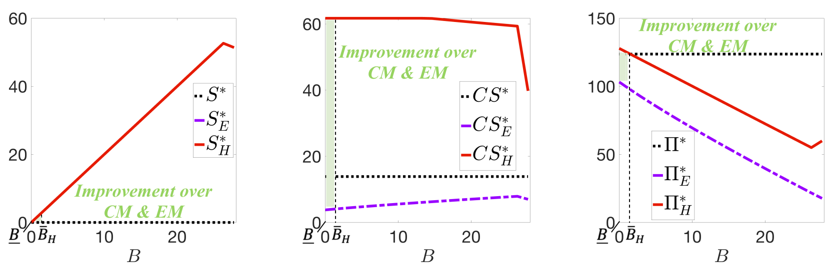
## OA3.1 Endogenous Service Prices

Note that for numerical studies we assume the distribution of consumer valuation $V$ is uniform. We define the consumer welfare as $CS = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t \lambda_t E[V - p_t | V \geq p_t]$, where $\lambda_t$ is the transaction volume in period $t$, $p_t$ is the service price, and the term $E[V - p_t | V \geq p_t]$ is the expected consumer surplus from a transaction.



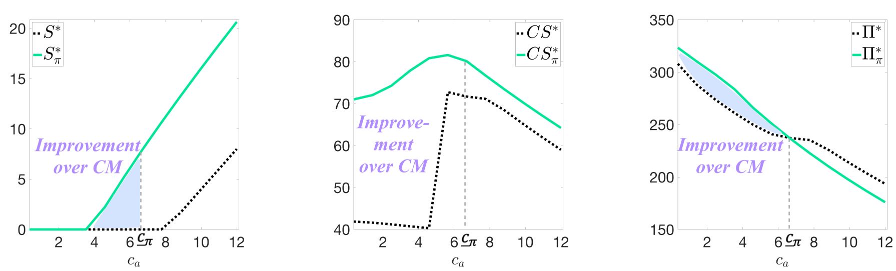

(a) EM: undercutting (b) C$^+$M: overjoining

**Figure OA.3 Issues of Uniform Classifications (Robust. Check - Endogenous Service Prices)**
($\beta_{\mathsf{h}} = 0.2, M_f = 2, M_a = 34, \gamma_{a,\mathsf{h}} = 0.8, \mu_{\mathsf{h},0} = 40, \mu_{\mathsf{l},0} = 15, \tau = 2, V \sim U[5, 50], r_0 = 9, c_f = 8, c_a = 16$)




(a) For Full-timers (b) For Consumers (c) For the Company

**Figure OA.4 Implications of the Hybrid Mode (Robust. Check - Endogenous Service Prices)**
($\beta_{\mathsf{h}} = 0.1, M_f = 2, M_a = 18, \gamma_{a,\mathsf{h}} = 0.6, \mu_{\mathsf{h},0} = 40, \mu_{\mathsf{l},0} = 25, \tau = 2, V \sim U[5, 50], r_0 = 8, c_f = 6, c_a = 14$)



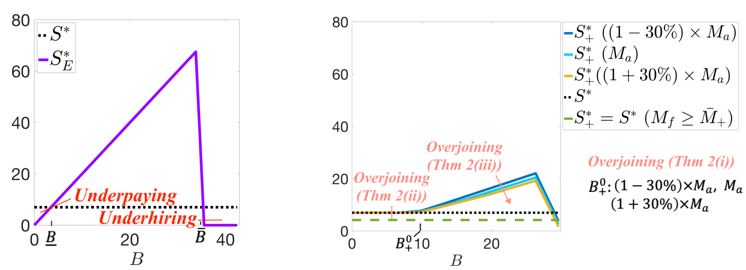

(a) For Full-timers (b) For Consumers (c) For the Company

**Figure OA.5 Implications of the Priority Mode (Robust. Check - Endogenous Service Prices)**
($\beta_{\mathsf{h}} = 0.2, M_f = 2, M_a = 18, \gamma_{a,\mathsf{h}} = 0.8, \mu_{\mathsf{h},0} = 40, \mu_{\mathsf{l},0} = 25, \tau = 2, V \sim U[5, 50], r_0 = 8, c_f = 6$)

The numerical results in Fig. OA.3 to Fig. OA.5 corroborate our key findings from the baseline; we still observe that uniform classifications may not enhance full-timers' welfare relative to the status quo of the CM due to either the undercutting issue in the EM or the overjoining issue in the C$^+$M, and that differentiated schemes (i.e., the HM and the C$^\pi$M) can improve over uniform approaches and the CM as well. In Section 7.1 we have also discussed a new finding in the C$^+$M: that is, contrary to Theorem 2, the C$^+$M may now hurt full-timers even for an intermediate benefit level $B$; see Fig. OA.3(b) for an illustration.

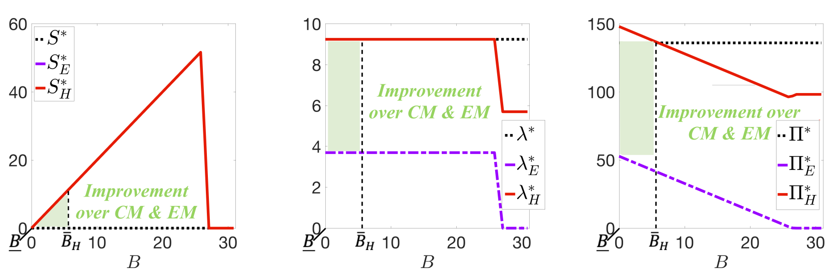
## OA3.2 Exact Closed Queueing Network Analysis

The wait time formulae we provide in Section 7.2.2 applies to both full- and part-timers in the CM, C$^+$M and HM and also to full-timers in the EM. For the priority mode C$^\pi$M, we first compute full-timers' average wait time $W^\pi_{f,t}$ and the overall wait time $W^\pi_t$ using the approach in Gross and Harris (1985), and then back out part-timers' wait time as $W^\pi_{a,t} = (W^\pi_t - \theta^\pi_f W^\pi_{f,t})/\theta^\pi_a$, where $\theta^\pi_f$ and $\theta^\pi_a$ denote the fractions of full- and part-timers in the aggregate worker arrival process, respectively; On the one hand, because full-timers' priority only changes the *order* of workers but not the *total* times workers being dispatched in any time unit, the overall wait time $W^\pi_t$, i.e., the average wait time between full- and part-timers, in the system can be computed using the same formulae for the FCFS scenario. On the other hand, because full-timers are prioritized, to them the whole system operates as if with no part-timer, and thus their average wait time $W^\pi_{f,t}$ can also be calculated using the FCFS formulae, with the total number of workers $M_t$ be replaced by the total of only full-timers $M_{f,t}$.




(a) EM: undercutting (b) C$^+$M: overjoining

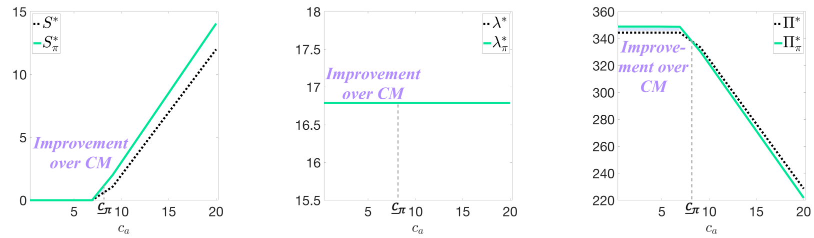
**Figure OA.6 Issues of Uniform Classifications (Robust. Check - Exact Closed Queueing Network Analysis)**
**($\beta_\mathsf{h} = 0.2, M_f = 2, M_a = 34, \gamma_{a,\mathsf{h}} = 0.8, \mu_\mathsf{h} = 40, \mu_\mathsf{l} = 25, \tau = 2, p_\mathsf{h} = 45, p_\mathsf{l} = 18, r_0 = 9, c_f = 8, c_a = 16$)**




(a) For Full-timers (b) For Consumers (c) For the Company

**Figure OA.7 Implications of the Hybrid Mode (Robust. Check - Exact Closed Queueing Network Analysis)**
**($\beta_\mathsf{h} = 0.1, M_f = 2, M_a = 8, \gamma_{a,\mathsf{h}} = 0.6, \mu_\mathsf{h} = 40, \mu_\mathsf{l} = 25, \tau = 2, p_\mathsf{h} = 45, p_\mathsf{l} = 18, r_0 = 12, c_f = 7, c_a = 8$)**

We now discuss how we derive $\theta^\pi_f$ and $\theta^\pi_a$, the fractions of full- and part-timers in the aggregate worker arrival process. Note that, the total times workers are dispatched in any time unit, or equivalently, the aggregate arrival rate of workers is $\lambda^\pi_t = M_t/(W^\pi_t + 1/\tau)$, and the arrival rate of




(a) For Full-timers (b) For Consumers (c) For the Company

**Figure OA.8 Implications of the Priority Mode (Robust. Check - Exact Closed Queueing Network Analysis)** **(**$\beta_{\mathsf{h}} = 0.2, M_f = 2, M_a = 18, \gamma_{a,\mathsf{h}} = 0.6, \mu_{\mathsf{h}} = 40, \mu_{\mathsf{l}} = 25, \tau = 2, p_{\mathsf{h}} = 45, p_{\mathsf{l}} = 18, r_0 = 8, c_f = 6$**)**

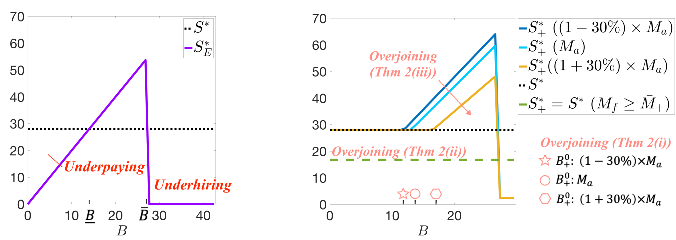
full-timers is $\lambda^\pi_{f,t} = M_{f,t}/(W^\pi_{f,t} + 1/\tau)$. The intuition is that, in a closed system with an average wait time $W$, the expected number of services a worker can complete is $1/(W + 1/\tau)$. Then given that there are $M$ workers in the system the total times workers arrive and get dispatched must be $M/(W + 1/\tau)$. The fractions of full- and part-timers in the arrival process are then given by $\theta^\pi_{f,t} = \lambda^\pi_{f,t}/\lambda^\pi_t$ and $\theta^\pi_{a,t} = 1 - \theta^\pi_{f,t}$, respectively, and we can back out part-timers' wait time $W^\pi_{a,t}$ using the equation $\theta^\pi_{f,t} W^\pi_{f,t} + \theta^\pi_{a,t} W^\pi_{a,t} = W^\pi_t$.

The numerical results in Fig. OA.6 to Fig. OA.8 corroborate our key findings from the baseline; we still observe that uniform classifications may not enhance full-timers' welfare relative to the status quo of the CM due to either the undercutting issue in the EM or the overjoining issue in the C$^+$M, and that differentiated schemes (i.e., the HM and the C$^\pi$M) can improve over uniform approaches and the CM as well.

### OA3.3 Alternative Dispatching Policy

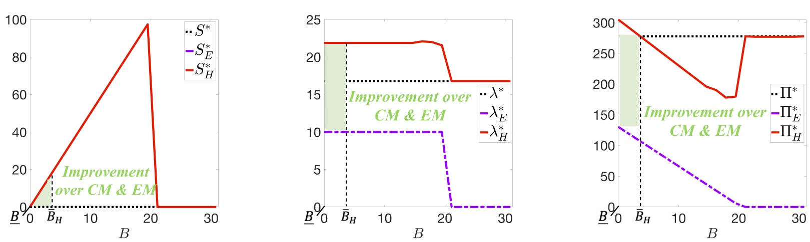
Similarly as in the extension with the exact closed-network formulation, here in the C$^\pi$M we also back out average wait time for part-timers as $W^\pi_{a,t} = (W^\pi_t - \theta^\pi_f W^\pi_{f,t})/\theta^\pi_a$, where $\theta^\pi_f$ and $\theta^\pi_a$ denote the fractions of full- and part-timers in the aggregate worker arrival process, respectively. Also note that, throughout the numerical analysis we set $\bar{e}$, the average pickup time when there is only one worker in a zone, to be 1/10.

The numerical results in Fig. OA.9 to Fig. OA.11 corroborate our key findings from the baseline; we still observe that uniform classifications may not enhance full-timers' welfare relative to the status quo of the CM due to either the undercutting issue in the EM or the overjoining issue in the C$^+$M, and that differentiated schemes (i.e., the HM and the C$^\pi$M) can improve over uniform approaches and the CM as well.



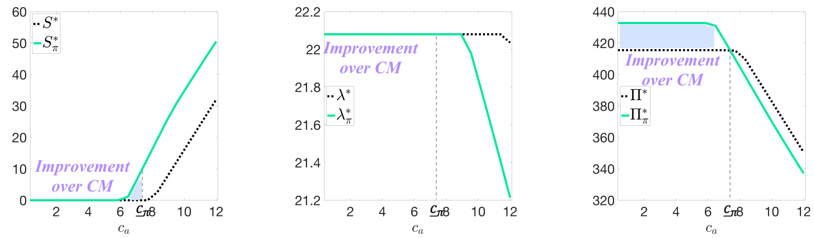

(a) EM: undercutting (b) C$^+$M: overjoining

**Figure OA.9** **Issues of Uniform Classifications (Robust. Check - Alternative Dispatching Policy)**
($\beta_h = 0.2, M_f = 2, M_a = 34, \gamma_{a,h} = 0.8, \mu_h = 40, \mu_l = 25, \tau = 2, p_h = 45, p_l = 18, r_0 = 9, c_f = 8, c_a = 16$)




(a) For Full-timers (b) For Consumers (c) For the Company

**Figure OA.10** **Implications of the Hybrid Mode (Robust. Check - Alternative Dispatching Policy)**
($\beta_h = 0.1, M_f = 5, M_a = 20, \gamma_{a,h} = 0.6, \mu_h = 40, \mu_l = 25, \tau = 2, p_h = 45, p_l = 18, r_0 = 12, c_f = 7, c_a = 8$)




(a) For Full-timers (b) For Consumers (c) For the Company

**Figure OA.11** **Implications of the Priority Mode (Robust. Check - Alternative Dispatching Policy)**
**(**$\beta_{\mathsf{h}} = 0.2, M_f = 4, M_a = 16, \gamma_{a,\mathsf{h}} = 0.6, \mu_{\mathsf{h}} = 40, \mu_{\mathsf{l}} = 25, \tau = 2, p_{\mathsf{h}} = 45, p_{\mathsf{l}} = 18, r_0 = 8, c_f = 6$**)**

## OA3.4 Dynamic Scheduling in the Employee Mode

In the baseline, we have assumed that the company will maintain a constant team of employees over the day. While this aligns with the literature on hybrid workforce (e.g., Milner and Pinker 2001, Dong and Ibrahim 2020), it admittedly misses the practice of dynamic scheduling in the service industry (see, e.g., Kamalahmadi et al. 2021). As such, we now let the company dynamically adjust the number of employees to serve on the platform period by period. Specifically, we assume that still only full-timers can be hired. For part-timers, while the company may offer part-time positions for them to fill, such positions are not likely to be as flexible as self-scheduled shifts in the contractor mode; for example, workers may no longer be able to choose to work only on some days in a week but not on the other, or even worse, they may have to "stand by" and respond to the company's urgent scheduling notices (Kamalahmadi et al. 2021). As such, we do not expect many part-timers to have a strong intension to fulfill those part-time positions. For simplicity, we assume that still part-timers will (mostly) exit the market.

Denote this alternative EM as E$^d$M. Further denote the workforce sizes for peak and off-peak periods as $\lambda_{E^d,\mathsf{h}}$ and $\lambda_{E^d,\mathsf{l}}$, respectively. One may interpret $\lambda_{E^d,\mathsf{l}}$ as the company's base service capacity throughout the day, and $\lambda_{E^d,\mathsf{h}} - \lambda_{E^d,\mathsf{l}}$ as part-time employees the company hires to better the demand in peak periods.[25,26] Note that, for full-timers who are hired only as part-time employees, they may not be eligible for all the employee benefits a full-time employee will be entitled to. As Yu et al. (2022) have highlighted, employees who work less than 20 hours a week may not get retirement benefits or health insurance. We thus assume that for these full-timers, the earning rate will be $r_{\alpha B} = r_0 + \alpha B$ for some discount $\alpha \in (0,1)$.

Given our setup above, the company's profit optimization problem in the E$^d$M is

$$\max_{(\lambda_{E^d,\mathsf{h}},\lambda_{E^d,\mathsf{l}})\in[0,\lambda_f]^2} \Pi_{E^d} \equiv \beta_\mathsf{h}(p_\mathsf{h} - r_{\alpha B}(\frac{1}{\mu_\mathsf{h} - \lambda_{E^d,\mathsf{h}}} + \frac{1}{\tau}))(\lambda_{E^d,\mathsf{h}} - \lambda_{E^d,\mathsf{l}}) + \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t(p_t - r_B(\frac{1}{\mu_t - \lambda_{E^d,t}} + \frac{1}{\tau}))\lambda_{E^d,\mathsf{l}}.$$

Denote by $\lambda^*_{E^d,\mathsf{h}}$ and $\lambda^*_{E^d,\mathsf{l}}$ the optimal workforce sizes. Further define $q^*_{E^d,t} = \lambda^*_{E^d,t}/\lambda_f$ as the fraction of full-timers in the labor pool $M_f$ that are hired in period $t \in \{\mathsf{h},\mathsf{l}\}$. We particularly focus one two outcomes measures in the equilibrium, full-timers' welfare $S^*_{E^d}$ and the company's optimal profit $\Pi^*_{E^d}$. In particular, full-timers' welfare is defined as $S^*_{E^d} = M_f \cdot T(q^*_{E^d,\mathsf{l}}B + (q^*_{E^d,\mathsf{h}} - q^*_{E^d,\mathsf{l}})\beta_\mathsf{h}\alpha B)$. We compare the outcomes in the E$^d$M and CM and highlight the following findings.

[25] We implicitly assume that full-timers who are hired only as part-time employees will be able to find another part-time job for the time they are not hired on the platform.

[26] Lemma OA.11 in the Appendix implies that the optimal service level in peak periods $\lambda^*_{E^d,\mathsf{h}}$ will indeed be higher than that in off-peak periods $\lambda^*_{E^d,\mathsf{l}}$ (i.e., $\lambda^*_{E^d,\mathsf{h}} - \lambda^*_{E^d,\mathsf{l}} \geq 0$).

Proposition 8 ($\mathrm{E}^d$M vs. CM). *Compare full-timers' welfare and the company's profit in the $E^d$M and CM, we have*

*(i) There exist* $\underline{B}_{E^d}, \overline{B}_{E^d}$ *such that if* $B \leq \underline{B}_{E^d}$ *or* $B \geq \overline{B}_{E^d}$*, we have* $S^*_{E^d} \leq S^*$*.*

*(ii) If the fraction of part-timers available in off-peak periods* $\gamma_{a,\mathsf{l}} \leq \overline{\gamma}_a$ *for some* $\overline{\gamma}_a$*, we have* $\Pi^* \geq \Pi^*_{E^d}$*.*

Proposition 8(i) implies that the *undercutting* issue we have identified in the EM persists in the $\mathrm{E}^d$M: full-timers still face being underpaid by the company when the benefit is sufficiently low (i.e., $B \leq \underline{B}_{E^d}$) or being underhired when the benefit is sufficiently high (i.e., $B \geq \overline{B}_{E^d}$).

For the company, recall that we show in Theorem 4 that the optimal profit in the CM $\Pi^*$ is always higher than that $\Pi^*_E$ in the EM (i.e., $\Pi^* \geq \Pi^*_E$). Now with the company being able to dynamically schedule in the $\mathrm{E}^d$M, Proposition 8(ii) implies that the profit premium in the CM requires an extra condition to hold: that the fraction of part-timers available in off-peak periods shall be relatively small (i.e., $\gamma_{a,\mathsf{l}} \leq \overline{\gamma}_a$).

To understand why the profit premium in the CM may evaporate (i.e., $\Pi^* \leq \Pi^*_E$) otherwise (i.e., $\gamma_{a,\mathsf{l}} > \overline{\gamma}_a$), first recall that in the EM the workforce is constant over time (i.e., $\lambda^*_{E,\mathsf{h}} = \lambda^*_{E,\mathsf{l}}$), and that the company makes a higher profit in the CM than in the EM thanks to two operational apparatuses, *dynamic capacity configuration* and *temporal incentive pooling*: on the one hand, the company can at least match the service level in the EM by nimbly sourcing the labor supply from full- and part-timers, and on the other hand the company can lower the total labor cost by differentiating the earning rates throughout the day, rather than maintaining a constant earning level. See Section 4.2 for more details.

Now, in the $\mathrm{E}^d$M the company dynamically schedules and the service level is higher in peak periods than in off-peak periods (i.e., $\lambda^*_{E^d,\mathsf{h}} \geq \lambda^*_{E^d,\mathsf{l}}$). To match the service levels in the $\mathrm{E}^d$M, the company in the CM shall generate a relatively high (low, respectively) earning rate $r_\mathsf{h}$ during peak ($r_\mathsf{l}$ off-peak, respectively) periods, otherwise there will be too few workers joining during peak periods (i.e., $\lambda^*_\mathsf{h} \leq \lambda^*_{E^d,\mathsf{h}}$) but too many of them joining in off-peak periods (i.e., $\lambda^*_\mathsf{l} \geq \lambda^*_{E^d,\mathsf{l}}$). However, to generate the high earning rate $r_\mathsf{h}$ and match the service level $\lambda^*_{E^d,\mathsf{h}}$ in the $\mathrm{E}^d$M during peak periods is more costly than to generate the same $r_\mathsf{h}$ but match the service level $\lambda^*_\mathsf{h}$ in the EM, since there are now more workers joining (i.e., $\lambda^*_{E^d,\mathsf{h}} \geq \lambda^*_\mathsf{h}$) and their utilization rate is lower. As a result, the total labor cost can be higher in the CM than that in the $\mathrm{E}^d$M.

While the company can lower the total labor cost by generating a lower earning rate during peak periods or even homogenizing the earning rates throughout the day, the service level in the CM will then become inefficiently low (high, respectively) during peak (off-peak, respectively) periods compared with that in the E$^d$M. In a nutshell, there is a tension between *dynamic capacity configuration* and *temporal incentive pooling*, and the company is no longer guaranteed to make a higher profit in the CM than in the E$^d$M.

Now, if the fraction of part-timers available in off-peak periods is relatively large (i.e., $\gamma_{a,\mathsf{l}} > \overline{\gamma}_a$), part-timers available in peak periods are relatively scarce. Chances are that the company may not be able to even match the service levels at all (particularly during peak periods). Even if the E$^d$M service levels can be attainted, to that end the company may still rely on differentiated earning rates $r_{\mathsf{h}} > r_{\mathsf{l}}$ to control workers' joining rates (particularly during off-peak periods given the relatively excessive number of part-timers). As such, the company may indeed end up with a lower profit than in the E$^d$M. See Fig. OA.12 for an illustration.

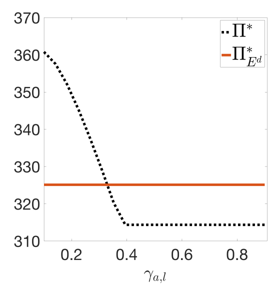



**Figure OA.12 Optimal Profit CM vs. E$^d$M**
*Note.* $\beta_{\mathsf{h}} = 0.2, M_f = 24, M_a = 16, \mu_{\mathsf{h}} = 45, \mu_{\mathsf{l}} = 15, \tau = 2, p_{\mathsf{h}} = 45, p_{\mathsf{l}} = 18, r_0 = 12, c_f = 8, c_a = 15$.

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## OA4. Proofs

Throughout the proofs we will drop the total time periods $T$ in full-timers' utility functions ($u = T\sum_{t\in\{\mathsf{h},\mathsf{l}\}} q_{f,t}(r_t - c_f)^+$), their outside option ($u_0 = T(r_0 - c_f)$), the transaction volume ($\lambda = T\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t\lambda_t$) and the company's profit ($\Pi = T\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t(p_t - w_t)\lambda_t$) for notational ease. Also, unless clarified otherwise, we will generally assume that the labor pool size is sufficiently small such that in both peak and off peak periods the gross arrival rates of full- and part-timers are less than the demand rates (i.e., $\mu_t > \lambda_f + \lambda_{a,t}$ for $t \in \{\mathsf{h},\mathsf{l}\}$). This is for the completeness of results: given our queueing-foundational setup, if in any period $t$ we have $\mu_t \le \lambda_f + \lambda_{a,t}$, then the company cannot let all workers participate as otherwise the system will be extremely overloaded. As such, to assume $\mu_t > \lambda_f + \lambda_{a,t}$ for $t \in \{\mathsf{h},\mathsf{l}\}$ is to include all possible cases of worker participation under consideration. In addition, expressions for monotonicity (i.e., increasing, decreasing) and magnitude comparison (i.e., higher/larger, lower/smaller) will be used in the non-strict sense unless (a) declared otherwise, and (b) when we are comparing workers' opportunity costs (ie.., full-timers' $c_f$ and part-timers $c_a$), as we have assumed in Section 3 that $c_f \neq c_a$.

**Proof of Lemma 1**

Recall the two-stage game in our model: full-timers decide whether to commit in stage 0, and then in stage 1 both committed full-timers and part-timers will decide whether to participate. Further define $\mathcal{Q}_q = (q_{f,\mathsf{h}}, q_{f,\mathsf{l}}, q_{a,\mathsf{h}}, q_{a,\mathsf{l}})$ as workers' participation equilibrium in stage 1 (or a *stage-1 equilibrium*) given that a fraction $q$ of all full-timers have committed in stage 0 and given piece-rate wages $(w_\mathsf{h}, w_\mathsf{l})$. We will establish the uniqueness of a market equilibrium $(\mathcal{Q}, \mathcal{R})$ in two steps.

**Step 1:** We show that given $w_\mathsf{h}$, $w_\mathsf{l}$ and full-timers' commitment probability $q$, there exists a unique *stage-1 equilibrium* $(\mathcal{Q}_q, \mathcal{R})$. Suppose instead the equilibrium is not unique in some period $t$. That is, there exist at least two nonidentical tuples $(q_{f,t}, q_{a,t}, r_t)$ and $(q'_{f,t}, q'_{a,t}, r'_t)$. By definition of equilibrium, we have

$$w_t = r_t\left(\frac{1}{\mu_t - q\cdot q_{f,t}\lambda_f - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}\right) = r'_t\left(\frac{1}{\mu_t - q\cdot q'_{f,t}\lambda_f - q'_{a,t}\lambda_{a,t}} + \frac{1}{\tau}\right) \qquad \text{(OA.1)}$$

Suppose $r_t > r'_t$. Then (OA.1) implies that $q\cdot q_{f,t}\lambda_f + q_{a,t}\lambda_{a,t} < q\cdot q'_{f,t}\lambda_f + q'_{a,t}\lambda_{a,t}$, which in turn implies that at least one of the following inequalities must hold: $q_{f,t} < q'_{f,t}$, and $q_{a,t} < q'_{a,t}$. Suppose first $q_{f,t} < q'_{f,t}$. This implies that $q_{f,t} \in [0,1)$, which requires the equilibrium earning rate $r_t = c_f$, i.e., full-timers' participation cost. But then $r'_t < r_t = c_f$, which renders $q'_{f,t} = 0$, contradicts. Then suppose $q_{a,t} < q'_{a,t}$. Similarly, this implies that $q_{a,t} \in [0,1)$ and $r_t = c_a$, yet we then have $r'_t < r_t = c_a$ and $q'_{a,t} = 0$. Again contradiction. Hence we must have $r_t = r'_t$ and thus $q_{f,t} = q'_{f,t}$ and $q_{a,t} = q'_{a,t}$. A similar argument will show that the case with $r_t < r'_t$ is impossible either.

**Step 2:** We now proceed to show that given $w_{\mathsf{h}}$, $w_{\mathsf{l}}$, there exists a unique equilibrium commitment probability $q$ among full-timers and thus, based on Step 1, a unique market equilibrium $(\mathcal{Q}, \mathcal{R})$. For conciseness, we will assume $c_a > c_f$. The case with $c_a < c_f$ can be verified similarly.

Suppose first the period-$t$ piece-rate wage $w_t \le c_f(1/(\mu_t - q\lambda_f) + 1/\tau)$. We claim that the only stage-1 equilibrium for period $t$ is $q_{f,t} = \max\{(\mu_t - 1/(w_t/c_f - 1/\tau))/(q\lambda_f), 0\}$, $q_{a,t} = 0$ and $r_t = c_f$. Any part-timer participating (i.e., $q_{a,t} > 0$) implies $r_t \ge c_a > c_f$, yet this in turn implies that all full-timers will participate (i.e., $q_{f,t} = 1$), which renders the average wage time $W_t = 1/(\mu_t - q\lambda_f - \lambda_{a,t})$ and thus the average earning rate $r_t = w_t/(W_t + 1/\tau) < c_f$. But then no full-timer will choose to participate (i.e., $q_{f,t} = 0$). Contradiction.

Then as $w_t$ migrates to $(c_f(1/(\mu_t - q\lambda_f) + 1/\tau), c_a(1/(\mu_t - q\lambda_f) + 1/\tau)]$, we have $q_{f,t} = 1$, $q_{a,t} = 0$, and $r_t = w_t/(1/(\mu_t - q\lambda_f) + 1/\tau) \in (c_f, c_a]$. And for $w_t \in (c_a(1/(\mu_t - q\lambda_f) + 1/\tau), c_a(1/(\mu_t - q\lambda_f - \lambda_{a,t}) + 1/\tau)]$, we have $q_{f,t} = 1$, $q_{a,t} = \max\{(\mu_t - q\lambda_f - 1/(w_t/c_a - 1/\tau))/\lambda_{a,t}, 0\}$, and $r_t = c_a$. Finally, for $w_t > c_a(1/(\mu_t - q\lambda_f - \lambda_{a,t}) + 1/\tau)$, we have $q_{f,t} = q_{a,t} = 1$ and $r_t = w_t/(1/(\mu_t - q\lambda_f - \lambda_{a,t}) + 1/\tau)$.

Define $\hat{q}^-_{a,t} = (\mu_t - \lambda_{a,t} - 1/(w_t/c_a - 1/\tau))/\lambda_f$, $\hat{q}_{a,t} = (\mu_t - 1/(w_t/c_a - 1/\tau))/\lambda_f$ and $\hat{q}_{f,t} = (\mu_t - 1/(w_t/c_f - 1/\tau))/\lambda_f$. By reverting the relation between the piece-rate wage $w_t$ and full-timers' commitment probability $q$, we summarize the average earning rate $r_t$ as follows:

$$r_t(q) = \begin{cases} w_t/(1/(\mu_t - q\lambda_f - \lambda_{a,t}) + 1/\tau) & \text{if } q \le \hat{q}^-_{a,t}, \\ c_a & \text{if } q \in [\hat{q}^-_{a,t}, \hat{q}_{a,t}], \\ w_t/(1/(\mu_t - q\lambda_f) + 1/\tau) & \text{if } q \in [\hat{q}_{a,t}, \hat{q}_{f,t}], \\ c_f & \text{otherwise.} \end{cases}$$

And similarly full- and part-timers' participation probabilities $q_{f,t}$ and $q_{a,t}$ as

$$q_{f,t}(q) = \begin{cases} 1 & \text{if } q \le \hat{q}_{f,t}, \\ \max\{\frac{\mu_t - 1/(w_t/c_f - 1/\tau)}{q\lambda_f}, 0\} & \text{otherwise} \end{cases}, \quad q_{a,t}(q) = \begin{cases} 1 & \text{if } q \le \hat{q}_{a,t}, \\ \max\{\frac{\mu_t - q\lambda_f - 1/(w_t/c_a - 1/\tau)}{\lambda_{a,t}}, 0\} & \text{otherwise.} \end{cases}$$

We illustrate $r_t(q)$ against $q$ in Fig OA.13.

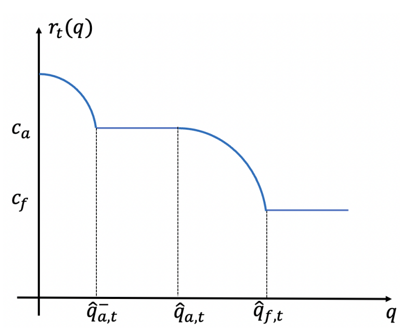


**Figure OA.13**

Observe that $r_t$ is decreasing in $q$ but also increasing in $w_t$. Hence full-timers' utility on the platform $u(q) = \beta_{\mathsf{h}}(r_{\mathsf{h}}(q) - c_f)^+ + \beta_{\mathsf{l}}(r_{\mathsf{l}}(q) - c_f)^+$ will decrease in $q$ but increase in $w_{\mathsf{h}}$ and $w_{\mathsf{l}}$. Recall that full-timers' outside option $u_0 \equiv r_0 - c_f$. By the definition of mixed strategy equilibrium, we shall have (i) $q=1$ if $u(1) \geq u_0$, (ii) $q=0$ if $u(0) \leq u_0$, and (iii) $q \in (0,1)$ and is determined by the equation $u(q) = u_0$ if $u_0 \in (u(1), u(0))$. The decreasing behavior of $u(q)$ and the assumptions that $c_a \neq r_0$ and $r_0 > c_f$ ensures the uniqueness of $q$. □

*Remark.* The Proof of Lemma 1 yields several results that be useful for verifying other results. See Section OA5.1 for details.

**Proof of Proposition 1**

**Proof of Proposition 1(i)** We first verify that $S^*_{fo} = 0$. As we note in Section 3, the full-timer-only CM is essentially a CM where part-timers' opportunity cost $c_a$ is sufficiently high that none of them will ever participate for any piece-rate wage or during any period. As such, the analysis we have completed so far for a general CM in the Proof of Lemma 1 and related also applies here.

Suppose instead $S^*_{fo} = q^*_{fo} M_f(u^*_{fo} - u_0) > 0$, where $q^*_{fo}$ denotes the equilibrium commitment probability among full-timers. This implies that $u^*_{fo} > u_0$ and, according to Lemma OA.8, that full-timers' committed probability $q^*_{fo} = 1$ and the existence of some period $t$ with the average earning rate $r^*_{fo,t} > r_0$ and committed full-timers' participation probability $q^{fo,*}_{f,t} = 1$. Then Lemma OA.1 implies that the optimal piece-rate wage the company has chosen for period $t$ must be $w^*_{fo,t} = r^*_{fo,t}(1/(\mu_t - \lambda_f) + 1/\tau)$. Yet this wage is obviously suboptimal: the company can make a strictly higher profit by instead paying $w^\epsilon_{fo,t} = (r^*_{fo,t} - \epsilon)(1/(\mu_t - \lambda_f) + 1/\tau)$ for some small $\epsilon > 0$ such that $r^*_t - \epsilon > r_0$, $q^* = 1$ and $q^{fo,t}_{f,t} = 1$; that is, the company can lower the wage without reducing the number of workers who choose to participate. Contradiction.

We now verify the conditions for $S^* > 0$. According to the proof for Proposition 1(iii) below, we have the following results.

LEMMA. *For $c_a \in (r_0, (r_0 - \beta_h c_f)/\beta_l)$, if $S^* > 0$, the company's optimal profit $\Pi^*$ must be*

$$\Pi^{*,+} \equiv \max_{(q_{a,h}, q_{a,l}) \in [0,1]^2} \sum_{t \in \{h,l\}} \beta_t (p_t - c_a(\frac{1}{\mu_t - \lambda_f - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}))(\lambda_f + q_{a,t}\lambda_{a,t}). \tag{OA.2}$$

*If instead $S^* = 0$ in the CM, the company's optimal profit $\Pi^*$ must be*

$$\begin{aligned}\Pi^{*,0} \equiv \max\{ & \max_{(q,q_{a,l}) \in [0,1]^2} \beta_h (p_h - \frac{r_0 - \beta_l c_a}{\beta_h}(\frac{1}{\mu_h - q\lambda_f} + \frac{1}{\tau}))q\lambda_f + \beta_l (p_l - c_a(\frac{1}{\mu_l - q\lambda_f - q_{a,l}\lambda_{a,l}} + \frac{1}{\tau}))(q\lambda_f + q_{a,l}\lambda_{a,l}), \\ & \max_{(q,q_{a,h}) \in [0,1]^2} \beta_h (p_h - c_a(\frac{1}{\mu_h - q\lambda_f - q_{a,h}\lambda_{a,h}} + \frac{1}{\tau}))(q\lambda_f + q_{a,h}\lambda_{a,h}) + \beta_l (p_l - \frac{r_0 - \beta_h c_a}{\beta_l}(\frac{1}{\mu_l - q\lambda_f} + \frac{1}{\tau}))q\lambda_f, \\ & \max_{r_h \in (c_a, \frac{r_0 - \beta_l c_f}{\beta_h}), q \in [0,1]} \beta_h (p_h - r_h(\frac{1}{\mu_h - q\lambda_f - \lambda_{a,h}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,h}) + \beta_l (p_l - \frac{r_0 - \beta_h r_h}{\beta_l}(\frac{1}{\mu_l - q\lambda_f} + \frac{1}{\tau}))q\lambda_f, \\ & \max_{(q,q_{f,l}) \in [0,1]^2} \beta_h (p_h - \frac{r_0 - \beta_l c_f}{\beta_h}(\frac{1}{\mu_h - q\lambda_f - \lambda_{a,h}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,h}) + \beta_l (p_l - c_f(\frac{1}{\mu_l - q_{f,l} \cdot q\lambda_f} + \frac{1}{\tau}))q_{f,l} \cdot q\lambda_f \}.\end{aligned}$$

PROOF. See Lemma OA.3 and the Proof. Full-timers' welfare is positive in the CM equilibrium (i.e., $u^* > u_0$) if and only if the optimal earning rates $r^*_{\mathsf{h}} = r^*_{\mathsf{l}} = c_a > r_0$. $\square$

We verify in the proof for Proposition 1(iii) that $\Pi^{*,+}$ is decreasing in $c_a$ while $\Pi^{*,0}$ is generally first decreasing and then increasing in $c_a$. As a result, there shall exist $\underline{c} \le \bar{c}$ such that if $c_a \in (\underline{c}, \bar{c})$ we have $\Pi^{*,+} > \Pi^{*,0}$— and thus the company will leave a positive surplus to full-timers in the CM equilibrium. $\square$

**Proof of Proposition 1(ii)**

By solving the program (OA.25), we derive the equilibrium transaction volume $(\lambda^*_{fo,\mathsf{h}}, \lambda^*_{fo,\mathsf{l}})$ in the full-timer-only CM as follows,

$$(\lambda^*_{fo,\mathsf{h}}, \lambda^*_{fo,\mathsf{l}}) = \begin{cases} (\lambda_f, \lambda_f) & \text{if } \lambda_f \le \mu_{\mathsf{l}} - \sqrt{\frac{\mu_{\mathsf{l}} \tau c_f}{p_{\mathsf{l}} \tau - c_f}} \\ (\min\{\lambda_f, \mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}} \tau r^{\ddagger}_{\mathsf{h}}}{p_{\mathsf{h}} \tau - r^{\ddagger}_{\mathsf{h}}}}\}, \mu_{\mathsf{l}} - \sqrt{\frac{\mu_{\mathsf{l}} \tau c_f}{p_{\mathsf{l}} \tau - c_f}}) & \text{otherwise} \end{cases}. \qquad \text{(OA.3)}$$

We now study the transaction volume in a general CM where both full- and part-timers may participate. Let $\bar{c}'$ defined in Proposition 1(ii) equal $r^{\ddagger}_{\mathsf{h}} \equiv (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$, and let part-timers' opportunity cost $c_a > \bar{c}'$. Lemma OA.2 in the Appendix OA.4 shows that the company's profit maximization with respect to piece-rate wages is equivalent to find out the optimal average earning rates as well as workers' participation probabilities. We show in Lemma OA.5 that when $c_a > \bar{c}'$ in the general CM the optimal average earning rates $(r^*_{\mathsf{h}}, r^*_{\mathsf{l}})$ must be one of the following options: $(c_a, c_a)$, $(c_a, c_f)$, $(r^{\ddagger}_{\mathsf{h}}, c_a)$, and $(r^{\ddagger}_{\mathsf{h}}, c_f)$; for any option, we can show that the corresponding transaction volume $\lambda^*_t$ will be higher than (OA.3) for any $t \in \{\mathsf{h}, \mathsf{l}\}$. Below we will elaborate with the case $(r^*_{\mathsf{h}}, r^*_{\mathsf{l}}) = (c_a, c_f)$. The other three cases can be verified similarly.

For $r^*_{\mathsf{h}} = c_a$ and $r^*_{\mathsf{l}} = c_f$, since $c_a > r^{\ddagger}_{\mathsf{h}}$, we have $u^* = \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t q^*_{f,t} (r^*_t - c_f)^+ > u_0$, which implies that full-timers' commitment probability $q = 1$. Also, according to the discussion in the Proof of Lemma 1, we must have $q^*_{f,\mathsf{h}} = 1$, $q^*_{a,\mathsf{h}} \in [0,1]$, $q^*_{f,\mathsf{h}} \in [0,1]$ and $q^*_{a,\mathsf{h}} = 0$. As such, the company's optimization problem must be

$$\max_{(q_{a,\mathsf{h}}, q_{f,\mathsf{l}}) \in [0,1]^2} \beta_{\mathsf{h}} (p_{\mathsf{h}} - c_a (\frac{1}{\mu_{\mathsf{h}} - \lambda_f - q_{a,\mathsf{h}} \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))(\lambda_f + q_{a,\mathsf{h}} \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}} (p_{\mathsf{l}} - c_f (\frac{1}{\mu_{\mathsf{l}} - q_{f,\mathsf{l}} \lambda_f} + \frac{1}{\tau})) q_{f,\mathsf{l}} \lambda_f.$$

Clearly we have $\lambda^*_{\mathsf{h}} = \lambda_f + q^*_{a,\mathsf{h}} \lambda_{a,\mathsf{h}} \ge \lambda^*_{fo,\mathsf{h}}$ and $\lambda^*_{\mathsf{l}} = \min\{\lambda_f, \mu_{\mathsf{l}} - \sqrt{\mu_{\mathsf{l}} \tau c_f / (p_{\mathsf{l}} \tau - c_f)}\} \ge \lambda^*_{fo,\mathsf{l}}$. $\square$

**Proof of Proposition 1(iii) Part I.** In the first part of the proof, we verify the existence of a $\bar{c}'''$ such that if part-timers' opportunity cost $c_a > \bar{c}'''$, we have $\Pi^* \ge \Pi^*_{fo}$.

Define $\bar{c}''' = (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}} > r_0 > c_f$. We show in Lemma OA.2 that the company's optimization problem in the CM is equivalent to choosing the optimal earning rates $\mathcal{R}$ and workers' participations $\mathcal{Q}$. When $c_a > \bar{c}'''$, a feasible solution for the company is to set the earning rate during peak periods

$r_{\mathsf{h}} = \bar{c}'''$ and then the earning rate during off-peak periods $r_{\mathsf{l}} = c_f$, both of which strictly lower than part-timers' opportunity cost $c_a$. Then according to Lemma OA.2, no part-timers will ever participate in any period (i.e., $q_{a,\mathsf{h}} = q_{a,\mathsf{l}} = 0$); full-timers will equilibrate among themselves with a mixed commitment strategy (i.e., $q \in [0,1]$), and committed full-timers will all participate during peak periods (i.e., $q_{f,\mathsf{h}} = 1$) but probabilistically do so during off-peak periods (i.e., $q_{f,\mathsf{l}} \in [0,1]$). The company's optimization in the CM is then

$$\max_{(q,q_{f,\mathsf{l}})\in[0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - \bar{c}'''(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_f(\frac{1}{\mu_{\mathsf{l}} - q_{f,\mathsf{l}} \cdot q\lambda_f} + \frac{1}{\tau}))q_{f,\mathsf{l}} \cdot q\lambda_f,$$

which is exactly the company's problem in the full-timer-only CM defined in (OA.25). As such, we must have $\Pi^* \geq \Pi^*_{fo}$. $\square$

**Proof of Proposition 1(iii) Part II.** In the second part of the proof, we verify the existence of $\underline{c}'' \leq \bar{c}''$ such that if part-timers' opportunity cost $c_a \in [\underline{c}'', \bar{c}'']$, we have $\Pi^* \leq \Pi^*_{fo}$.

To proceed, consider $c_a \in (r_0, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$. The key to the proof is to show that under such a condition the company's profit $\Pi$ will first decrease but then increase in $c_a$. To this end, below we will go over all earning rates that are potentially optimal (specified in Lemma OA.3) and study the structural property of the company's profit function should any pair of such earning rates be optimal indeed.

*Case I: $r_h = r_l = c_a$.* In this case the company's profit $\Pi^*$ will be decreasing in $c_a$. To see this, both in peak and off-peak periods part-timers will participate with mixed strategies while committed full-timers will always participate; in particular, full-timers in this case will all commit to working on the platform as the expected utility is strictly higher than their outside option (i.e., $u = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (r_t - c_f)^+ > u_0$). The company's optimization problem is thus to control part-timers' participation probabilities $q_{a,\mathsf{h}}$ and $q_{a,\mathsf{l}}$, i.e.,

$$\max_{(q_{a,\mathsf{h}},q_{a,\mathsf{l}})\in[0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \lambda_f - q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))(\lambda_f + q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda_f - q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}} + \frac{1}{\tau}))(\lambda_f + q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}),$$

which, as one can verify using the Envelope theorem, will decrease in $c_a$.

*Case II: $r_h = (r_0 - \beta_l c_a)/\beta_h, r_l = c_a$.* We claim that the company's profit $\Pi^*$ will decrease in $c_a$ in this case. To see this, first notice that $(r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}} \in (c_f, c_a)$ given that $c_a \in (r_0, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$. As such, during peak periods all committed full-timers will participate but no part-timers will do so. Similarly, because $r_{\mathsf{l}} = c_a$, during off-peak periods still all committed full-timers will participate yet this time part-timers will participate with a mixed strategy. Also note that full-timers will commit with a mixed strategy since the expected utility on the platform exactly equals their outside option (i.e., $u=\sum_{t\in\{\mathsf{h},\mathsf{l}\}}\beta_t(r_t-c_f)^+=u_0$). Hence, the company's optimization problem is thus to control full-timers' commitment probability $q$ and part-timers' participation probability $q_{a,\mathsf{l}}$, i.e.,

$$\max_{(q,q_{a,\mathsf{l}})\in[0,1]^2}\beta_\mathsf{h}(p_\mathsf{h}-\frac{r_0-\beta_\mathsf{l}c_a}{\beta_\mathsf{h}}(\frac{1}{\mu_\mathsf{h}-q\lambda_f}+\frac{1}{\tau}))q\lambda_f+\beta_\mathsf{l}(p_\mathsf{l}-c_a(\frac{1}{\mu_\mathsf{l}-q\lambda_f-q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}}+\frac{1}{\tau}))(q\lambda_f+q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}).$$

By applying the Envelope theorem with respect to $c_a$, we have

$$\begin{aligned}\frac{d\Pi^*}{dc_a}&=(\frac{1}{\mu_\mathsf{h}-q\lambda_f}+\frac{1}{\tau})q\lambda_f-(\frac{1}{\mu_\mathsf{l}-q\lambda_f-q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}}+\frac{1}{\tau})(q\lambda_f+q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}})\\&=\frac{-q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}}{\tau}+\frac{q\lambda_f}{\mu_\mathsf{h}-q\lambda_f}-\frac{q\lambda_f+q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}}{\mu_\mathsf{l}-q\lambda_f-q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}}<0,\end{aligned}$$

which implies that the company's optimal profit will decrease in $c_a$.

*Case III:* $r_h=c_a, r_l=(r_0-\beta_hc_a)/\beta_l$. We claim that in this case the company's profit $\Pi^*$ will either (1) increase, or (2) decrease, or (3) first decrease but then increase in $c_a$. To see this, given $r_\mathsf{h}$ and $r_\mathsf{l}$, similarly as in Case III one can verify that (1) during peak periods, all committed full-timers will participate but part-timers will participate with a mixed strategy, (2) during off-peak periods still all committed full-timers will participate but no part-timers will do so, and (3) full-timers will commit with a mixed strategy since the expected utility on the platform exactly equals their outside option. Hence, the company's optimization problem is thus to control full-timers' commitment probability $q$ and part-timers' participation probability $q_{a,\mathsf{h}}$, i.e.,

$$\max_{(q,q_{a,\mathsf{h}})\in[0,1]^2}\beta_\mathsf{h}(p_\mathsf{h}-c_a(\frac{1}{\mu_\mathsf{h}-q\lambda_f-q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}}+\frac{1}{\tau})\ )(\ q\lambda_f+q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}})+\beta_\mathsf{l}(p_\mathsf{l}-\frac{r_0-\beta_\mathsf{h}c_a}{\beta_\mathsf{l}}(\frac{1}{\mu_\mathsf{l}-q\lambda_f}+\frac{1}{\tau}))q\lambda_f.\quad\text{(OA.4)}$$

For this optimization problem, given any $q>0$, the optimization with respect to $q_{a,\mathsf{h}}$ yields the following solution,

$$q^*_{a,\mathsf{h}}=\min\{1,\frac{(\mu_\mathsf{h}-\sqrt{\frac{\mu_\mathsf{h}\tau c_a}{p_\mathsf{h}\tau-c_a}}-q\lambda_f)^+}{\lambda_{a,\mathsf{h}}}\},$$

where $x^+=\max\{x,0\}$. Define $\bar{\lambda_\mathsf{h}}=\mu_\mathsf{h}-\sqrt{\mu_\mathsf{h}\tau c_a/(p_\mathsf{h}\tau-c_a)}$ and $\bar{\lambda_\mathsf{h}}'=\mu_\mathsf{h}-\sqrt{\mu_\mathsf{h}\tau c_a/(p_\mathsf{h}\tau-c_a)}-\lambda_{a,\mathsf{h}}$. Plugging $q^*_{a,\mathsf{h}}$ into the problem (OA.4), we have

$$\begin{aligned}\Pi^*=\max\{\quad&\max_{q\lambda_f\le\bar{\lambda_\mathsf{h}}'}\quad\Pi_1\equiv\beta_\mathsf{h}(p_\mathsf{h}-c_a(\frac{1}{\mu_\mathsf{h}-q\lambda_f-\lambda_{a,\mathsf{h}}}+\frac{1}{\tau}))(q\lambda_f+\lambda_{a,\mathsf{h}})\\&+\quad\beta_\mathsf{l}(p_\mathsf{l}-\frac{r_0-\beta_\mathsf{h}c_a}{\beta_\mathsf{l}}(\frac{1}{\mu_\mathsf{l}-q\lambda_f}+\frac{1}{\tau}))q\lambda_f,\\&\max_{q\lambda_f\in[\bar{\lambda_\mathsf{h}}',\bar{\lambda_\mathsf{h}}]}\Pi_2\equiv\beta_\mathsf{h}(p_\mathsf{h}-c_a(\sqrt{\frac{p_\mathsf{h}\tau-c_a}{\mu_\mathsf{h}\tau c_a}}+\frac{1}{\tau}))(\mu_\mathsf{h}-\sqrt{\frac{\mu_\mathsf{h}\tau c_a}{p_\mathsf{h}\tau-c_a}})\end{aligned}\qquad\text{(OA.5)}$$

$$+ \quad \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f. \tag{OA.6}$$

$$\max_{q\lambda_f \geq \bar{\lambda}_{\mathsf{h}}} \quad \Pi_3 \equiv \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f$$

$$+ \quad \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f.\} \tag{OA.7}$$

Further define $\bar{\lambda}_{\mathsf{l}} = \mu_{\mathsf{l}} - \sqrt{\mu_{\mathsf{l}}\tau r_{\mathsf{l}}/(p_{\mathsf{l}}\tau - r_{\mathsf{l}})}$. To understand the impact of $c_a$ on $\Pi^*$, a key step is to pin down the optimal $q^*$ for $\Pi^*$, which in turn hinges on the structural properties of $\Pi_1$ defined in (OA.5) and $\Pi_2$ defined in (OA.6). Note that, we have

$$\frac{d\Pi_1}{dq} \propto \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - q\lambda_f - \lambda_{a,\mathsf{h}})^2} + \frac{1}{\tau})) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - q\lambda_f)^2} + \frac{1}{\tau})), \tag{OA.8}$$

$$\frac{d\Pi_2}{dq} \propto p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - q\lambda_f)^2} + \frac{1}{\tau}), \tag{OA.9}$$

$$\frac{d\Pi_3}{dq} \propto \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - q\lambda_f)^2} + \frac{1}{\tau})) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - q\lambda_f)^2} + \frac{1}{\tau})), \tag{OA.10}$$

where the notation $x \propto y$ implies that $x = ky$ for some irrelevant constant $k$. One may notice that all the derivatives above are decreasing in $q$; in other words, all of $\Pi_1$, $\Pi_2$ and $\Pi_3$ are concave in $q$. In addition, $q\lambda_f = \bar{\lambda}_{\mathsf{h}}{}'$ and $q\lambda_f = \bar{\lambda}_{\mathsf{l}}$ are roots for the first and the second summand in (OA.8), and $q\lambda_f = \bar{\lambda}_{\mathsf{h}}$ is the root for the first summand in (OA.10), respectively. We proceed with the following three scenarios.

*Scenario A:* $\bar{\lambda}_l \geq \bar{\lambda}_h$. First, for $q\lambda_f \leq \bar{\lambda}_{\mathsf{h}}{}'$, one can verify that $d\Pi/dq = d\Pi_1/dq > 0$ for all $q\lambda_f \leq \bar{\lambda}_{\mathsf{h}}{}'$, and this implies that the locally optimal (i.e., for $q\lambda_f \leq \bar{\lambda}_{\mathsf{h}}{}'$) commitment probability $q^*_{loc}$ must be $q^*_{loc}\lambda_f = \bar{\lambda}_{\mathsf{h}}{}'$. We thus have

$$\Pi_1^* \equiv \max_{q\lambda_f \leq \bar{\lambda}_{\mathsf{h}}{}'} \Pi_1 = \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\sqrt{\frac{p_{\mathsf{h}}\tau - c_a}{\mu_{\mathsf{h}}\tau c_a}} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}})$$
$$+ \quad \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - (\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}} - \lambda_{a,\mathsf{h}})} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}} - \lambda_{a,\mathsf{h}})$$

Next, for $q\lambda_f \in [\bar{\lambda}_{\mathsf{h}}{}', \bar{\lambda}_{\mathsf{h}}]$, because $\lambda_{\mathsf{h}} \leq \lambda_{\mathsf{l}}$, one can verify that $d\Pi/dq = d\Pi_2/dq > 0$ for all $q\lambda_f \leq [\bar{\lambda}_{\mathsf{h}}{}', \bar{\lambda}_{\mathsf{h}}]$, and this indicates that the locally optimal (i.e., for $q\lambda_f \in [\bar{\lambda}_{\mathsf{h}}{}', \bar{\lambda}_{\mathsf{h}}]$) commitment probability $q^*_{loc}$ must be $q^*_{loc}\lambda_f = \bar{\lambda}_{\mathsf{h}}$. Hence,

$$\Pi_2^* \equiv \max_{q\lambda_f \in [\bar{\lambda}_{\mathsf{h}}{}', \bar{\lambda}_{\mathsf{h}}]} \Pi_2 = \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\sqrt{\frac{p_{\mathsf{h}}\tau - c_a}{\mu_{\mathsf{h}}\tau c_a}} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}})$$
$$+ \quad \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - (\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}})} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}}),$$

which is *higher* than $\max_{q\lambda_f \leq \bar{\lambda}_{\mathsf{h}}{}'} \Pi_1$ given the concavity of $\Pi_2$.

Finally, for $q\lambda_f \ge \bar{\lambda}_\mathsf{h}$, one can verify that $d\Pi/dq = d\Pi_3/dq > 0$ at $q\lambda_f = \bar{\lambda}_\mathsf{h}$ and $d\Pi/dq < 0$ at $q\lambda_f = \bar{\lambda}_\mathsf{l}$. With the concavity of $\Pi_3$, this indicates that the locally optimal (i.e., for $q\lambda_f \ge \bar{\lambda}_\mathsf{h}$) commitment probability $q^*_{loc}$ must be such that $d\Pi/dq = 0$ at $q = q^*_{loc}$, and this $\lambda^\star_f \equiv q^*_{loc}\lambda_f \in (\bar{\lambda}_\mathsf{h}, \bar{\lambda}_\mathsf{l})$. We thus have

$$\Pi^*_3 \equiv \max_{q\lambda_f \ge \bar{\lambda}_\mathsf{h}} \Pi_3 = \beta_\mathsf{h}(p_\mathsf{h} - c_a(\frac{1}{\mu_\mathsf{h} - \lambda^\star_f} + \frac{1}{\tau}))\lambda^\star_f + \beta_\mathsf{l}(p_\mathsf{l} - \frac{r_0 - \beta_\mathsf{h} c_a}{\beta_\mathsf{l}}(\frac{1}{\mu_\mathsf{l} - \lambda^\star_f} + \frac{1}{\tau}))\lambda^\star_f,$$

which is *higher* than $\max_{q\lambda_f \in [\bar{\lambda}_\mathsf{h}', \bar{\lambda}_\mathsf{h}]} \Pi_2$ because of the concavity of $\Pi_3$. Therefore, the (globally) optimal commitment probability $q^*$ for $\Pi^*$ must be $q^*\lambda_f = \lambda^\star_f$.

With $q^*$ and $q^*_{a,\mathsf{h}}$, we can derive the transaction volume in peak and off-peak periods respectively as $\lambda^*_\mathsf{h} = \lambda^*_\mathsf{l} = \lambda^\star_f$. Applying the Envelope theorem to $\Pi^*$ defined in (OA.4) with respect to $c_a$, we have

$$\frac{d\Pi^*}{dc_a} = -(\frac{1}{\mu_\mathsf{h} - \lambda^*_\mathsf{h}} + \frac{1}{\tau})\lambda^*_\mathsf{h} + (\frac{1}{\mu_\mathsf{l} - \lambda^*_\mathsf{l}} + \frac{1}{\tau})\lambda^*_\mathsf{l} > 0,$$

which implies the optimal profit $\Pi^*$ will increase in $c_a$.

*Scenario B:* $\bar{\lambda}_l \in [\bar{\lambda}_h', \bar{\lambda}_h]$. We claim that in this subcase the optimal $q^*$ for $\Pi^*$ will be $\bar{\lambda}_\mathsf{l}$. To see this, first, for $q\lambda_f \le \bar{\lambda}_\mathsf{h}'$, one can verify that $d\Pi/dq = d\Pi_1/dq > 0$ for all $q\lambda_f \le \bar{\lambda}_\mathsf{h}'$, and this implies that the locally optimal (i.e., for $q\lambda_f \le \bar{\lambda}_\mathsf{h}'$) commitment probability $q^*_{loc}$ must be just $q^*_{loc}\lambda_f = \bar{\lambda}_\mathsf{h}'$. We thus have

$$\begin{aligned}\Pi^*_1 = \max_{q\lambda_f \le \bar{\lambda}_\mathsf{h}'} \Pi_1 = {} & \beta_\mathsf{h}(p_\mathsf{h} - c_a(\sqrt{\frac{p_\mathsf{h}\tau - c_a}{\mu_\mathsf{h}\tau c_a}} + \frac{1}{\tau}))(\mu_\mathsf{h} - \sqrt{\frac{\mu_\mathsf{h}\tau c_a}{p_\mathsf{h}\tau - c_a}}) \\ & + \beta_\mathsf{l}(p_\mathsf{l} - \frac{r_0 - \beta_\mathsf{h} c_a}{\beta_\mathsf{l}}(\frac{1}{\mu_\mathsf{l} - (\mu_\mathsf{h} - \sqrt{\frac{\mu_\mathsf{h}\tau c_a}{p_\mathsf{h}\tau - c_a}} - \lambda_{a,\mathsf{h}})} + \frac{1}{\tau}))(\mu_\mathsf{h} - \sqrt{\frac{\mu_\mathsf{h}\tau c_a}{p_\mathsf{h}\tau - c_a}} - \lambda_{a,\mathsf{h}})\end{aligned}$$

Next, for $q\lambda_f \in [\bar{\lambda}_\mathsf{h}', \bar{\lambda}_\mathsf{h}]$, since now we have $\bar{\lambda}_\mathsf{l} \in [\bar{\lambda}_\mathsf{h}', \bar{\lambda}_\mathsf{h}]$, one can verify that $d\Pi/dq = d\Pi_2/dq \ge 0$ for $q\lambda_f \le \bar{\lambda}_\mathsf{l}$ and $d\Pi/dq < 0$ otherwise. With the concavity of $\Pi_2$, this indicates that the locally optimal (i.e., for $q\lambda_f \in [\bar{\lambda}_\mathsf{h}', \bar{\lambda}_\mathsf{h}]$) commitment probability $q^*_{loc}$ must be $q^*_{loc}\lambda_f = \bar{\lambda}_\mathsf{l}$. Hence,

$$\begin{aligned}\Pi^*_2 = \max_{q\lambda_f \in [\bar{\lambda}_\mathsf{h}', \bar{\lambda}_\mathsf{h}]} \Pi_2 = {} & \beta_\mathsf{h}(p_\mathsf{h} - c_a(\sqrt{\frac{p_\mathsf{h}\tau - c_a}{\mu_\mathsf{h}\tau c_a}} + \frac{1}{\tau}))(\mu_\mathsf{h} - \sqrt{\frac{\mu_\mathsf{h}\tau c_a}{p_\mathsf{h}\tau - c_a}}) \\ & + \beta_\mathsf{l}(p_\mathsf{l} - \frac{r_0 - \beta_\mathsf{h} c_a}{\beta_\mathsf{l}}(\sqrt{\frac{p_\mathsf{l}\tau - r_\mathsf{l}}{\mu_\mathsf{l}\tau r_\mathsf{l}}} + \frac{1}{\tau}))(\mu_\mathsf{l} - \sqrt{\frac{\mu_\mathsf{l}\tau r_\mathsf{l}}{p_\mathsf{l}\tau - r_\mathsf{l}}}),\end{aligned}$$

which is *higher* than $\max_{q\lambda_f \le \bar{\lambda}_\mathsf{h}'} \Pi_1$ because of the concavity of $\Pi_2$.

Finally, for $q\lambda_f \ge \bar{\lambda}_\mathsf{h}$, one can verify that $d\Pi/dq = d\Pi_3/dq \le 0$ for any $q$ such that $q\lambda_f \ge \bar{\lambda}_\mathsf{h}$. Hence the locally optimal (i.e., for $q\lambda_f > \bar{\lambda}_\mathsf{h}$) commitment probability $q^*_{loc}$ must be $q^*_{loc}\lambda_f = \bar{\lambda}_\mathsf{h}$ and

$$\Pi^*_3 = \max_{q\lambda_f \ge \bar{\lambda}_\mathsf{h}} \Pi_3 = \beta_\mathsf{h}(p_\mathsf{h} - c_a(\sqrt{\frac{p_\mathsf{h}\tau - c_a}{\mu_\mathsf{h}\tau c_a}} + \frac{1}{\tau}))(\mu_\mathsf{h} - \sqrt{\frac{\mu_\mathsf{h}\tau c_a}{p_\mathsf{h}\tau - c_a}})$$

$$+ \quad \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - (\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}})} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}}),$$

which is clearly *lower* than $\max_{q\lambda_f \in [\bar{\lambda_{\mathsf{h}}}', \bar{\lambda_{\mathsf{h}}}]} \Pi_2$. Therefore, the (globally) optimal commitment probability $q^*$ for $\Pi^*$ must be $q^*\lambda_f = \bar{\lambda_{\mathsf{l}}} \in [\bar{\lambda_{\mathsf{h}}}', \bar{\lambda_{\mathsf{h}}}]$.

Now, with $q^*$ and $q^*_{a,\mathsf{h}}$, we can derive the transaction volume in peak and off-peak periods respectively as

$$\lambda^*_{\mathsf{h}} = \mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}}, \lambda^*_{\mathsf{l}} = \mu_{\mathsf{l}} - \sqrt{\frac{\mu_{\mathsf{l}}\tau r_{\mathsf{l}}}{p_{\mathsf{l}}\tau - r_{\mathsf{l}}}}.$$

By applying the Envelope theorem to $\Pi^*$ defined in (OA.4) with respect to $c_a$, we have

$$\frac{d\Pi^*}{dc_a} = -(\frac{1}{\mu_{\mathsf{h}} - \lambda^*_{\mathsf{h}}} + \frac{1}{\tau})\lambda^*_{\mathsf{h}} + (\frac{1}{\mu_{\mathsf{l}} - \lambda^*_{\mathsf{l}}} + \frac{1}{\tau})\lambda^*_{\mathsf{l}},$$

which will increase in $c_a$ because (1) $\lambda^*_{\mathsf{h}}$ is decreasing in $c_a$, and (2) $\lambda^*_{\mathsf{l}}$ will increase in $c_a$ given that $r_{\mathsf{l}}$ is decreasing in $c_a$. Hence, it is possible that (1) $d\Pi^*/dc_a > 0$ (and thus $\Pi^*$ is increasing in $c_a$), (2) $d\Pi^*/dc_a < 0$ (and thus $\Pi^*$ is decreasing in $c_a$), or (3) $d\Pi^*/dc_a$ is first negative but then positive (and thus $\Pi^*$ is first decreasing but then increasing in $c_a$).

*Scenario C:* $\bar{\lambda_l} < \bar{\lambda_h}'$. For $q\lambda_f \leq \bar{\lambda_{\mathsf{h}}}'$, one can verify that $d\Pi/dq = d\Pi_1/dq > 0$ at $q\lambda_f = \bar{\lambda_{\mathsf{l}}}$ and $d\Pi/dq < 0$ at $q\lambda_f = \bar{\lambda_{\mathsf{h}}}'$. With the concavity of $\Pi_1$, this indicates that the locally optimal (i.e., for $q\lambda_f \leq \bar{\lambda_{\mathsf{h}}}'$) commitment probability $q^*_{loc}$ must be such that $d\Pi/dq = 0$ at $q = q^*_{loc}$, and this $\lambda^\star_f \equiv q^*_{loc}\lambda_f \in (\bar{\lambda_{\mathsf{l}}}, \bar{\lambda_{\mathsf{h}}}')$. We thus have

$$\Pi^*_1 = \max_{q\lambda_f \leq \bar{\lambda_{\mathsf{h}}}'} \Pi_1 = \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \lambda^\star_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))(\lambda^\star_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - \lambda^\star_f} + \frac{1}{\tau}))\lambda^\star_f.$$

Next, for $q\lambda_f \in [\bar{\lambda_{\mathsf{h}}}', \bar{\lambda_{\mathsf{h}}}]$, since now we have $\bar{\lambda_{\mathsf{l}}} < \bar{\lambda_{\mathsf{h}}}'$, one can verify that $d\Pi/dq = d\Pi_2/dq \leq 0$ for all $q\lambda_f \geq \bar{\lambda_{\mathsf{l}}}'$. With the concavity of $\Pi_2$, this indicates that the locally optimal (i.e., for $q\lambda_f \in [\bar{\lambda_{\mathsf{h}}}', \bar{\lambda_{\mathsf{h}}}]$) commitment probability $q^*_{loc}$ must be $q^*_{loc}\lambda_f = \bar{\lambda_{\mathsf{h}}}'$. Hence,

$$\begin{aligned}\Pi^*_2 = \max_{q\lambda_f \in [\bar{\lambda_{\mathsf{h}}}', \bar{\lambda_{\mathsf{h}}}]} \Pi_2 = &\beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\sqrt{\frac{p_{\mathsf{h}}\tau - c_a}{\mu_{\mathsf{h}}\tau c_a}} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}})\\ &+ \quad \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - (\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}} - \lambda_{a,\mathsf{h}})} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}} - \lambda_{a,\mathsf{h}}),\end{aligned}$$

which is clearly *lower* than $\max_{q\lambda_f \leq \bar{\lambda_{\mathsf{h}}}'} \Pi_1$.

Finally, for $q\lambda_f \geq \bar{\lambda_{\mathsf{h}}}$, one can verify that $d\Pi/dq = d\Pi_3/dq \leq 0$ for any $q$ such that $q\lambda_f \geq \bar{\lambda_{\mathsf{h}}}$. Hence the locally optimal (i.e., for $q\lambda_f > \bar{\lambda_{\mathsf{h}}}$) commitment probability $q^*_{loc}$ must be $q^*_{loc}\lambda_f = \bar{\lambda_{\mathsf{h}}}$ and

$$\Pi^*_3 = \max_{q\lambda_f \geq \bar{\lambda_{\mathsf{h}}}} \Pi_3 = \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\sqrt{\frac{p_{\mathsf{h}}\tau - c_a}{\mu_{\mathsf{h}}\tau c_a}} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}})$$

$$+ \quad \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - (\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}})} + \frac{1}{\tau}))(\mu_{\mathsf{h}} - \sqrt{\frac{\mu_{\mathsf{h}}\tau c_a}{p_{\mathsf{h}}\tau - c_a}}),$$

which is clearly *lower* than $\max_{q\lambda_f \in [\bar{\lambda_{\mathsf{h}}}', \bar{\lambda_{\mathsf{h}}}]} \Pi_2$. Therefore, the (globally) optimal commitment probability $q^*$ for $\Pi^*$ must be $q^*\lambda_f = \lambda_f^\star \in (\bar{\lambda_{\mathsf{l}}}, \bar{\lambda_{\mathsf{h}}}')$.

Now, with $q^*$ and $q^*_{a,\mathsf{h}}$, we can derive the transaction volume in peak and off-peak periods respectively as

$$\lambda_{\mathsf{h}}^* = \lambda_f^\star + \lambda_{a,\mathsf{h}}, \lambda_{\mathsf{l}}^* = \lambda_f^\star.$$

By applying the Envelope theorem to $\Pi^*$ defined in (OA.4) with respect to $c_a$, we have

$$\begin{aligned} \frac{d\Pi^*}{dc_a} &= -(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}^*} + \frac{1}{\tau})\lambda_{\mathsf{h}}^* + (\frac{1}{\mu_{\mathsf{l}} - \lambda_{\mathsf{l}}^*} + \frac{1}{\tau})\lambda_{\mathsf{l}}^*, \\ \Rightarrow \frac{d^2\Pi^*}{dc_a^2} &= (\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - \lambda_{\mathsf{l}}^*)^2} - \frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}^*)^2})\frac{dq^*}{dc_a}. \end{aligned} \tag{OA.11}$$

Recall the first-order derivative defined in (OA.8). For $q = q^*$, we have

$$\frac{d^2\Pi}{dqdc_a} = \frac{d^2\Pi_1}{dqdc_a} \propto -\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \lambda_{\mathsf{h}})^2} + \frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - \lambda_{\mathsf{l}})^2},$$

which implies that $q^*$ will increase in $c_a$ and thus $dq^*/dc_a \geq 0$ if and only if $-\mu_{\mathsf{h}}/(\mu_{\mathsf{h}} - \lambda_{\mathsf{h}})^2 + \mu_{\mathsf{l}}/(\mu_{\mathsf{l}} - \lambda_{\mathsf{l}})^2 \geq 0$. Therefore, $d^2\Pi^*/dc_a^2$ defined in (OA.11) must be positive, which implies that $d\Pi^*/dc_a$ will increase in $c_a$. Hence, as with Scenario B, it is possible that (1) $d\Pi^*/dc_a > 0$ (and thus $\Pi^*$ is increasing in $c_a$), (2) $d\Pi^*/dc_a < 0$ (and thus $\Pi^*$ is decreasing in $c_a$), or (3) $d\Pi^*/dc_a$ is first negative but then positive (and thus $\Pi^*$ is first decreasing but then increasing in $c_a$).

*Case IV: $r_h \in (c_a, (r_0 - \beta_l c_f)/\beta_h), r_l = (r_0 - \beta_h r_h)/\beta_l$.* Given that $r_{\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}})/\beta_{\mathsf{l}} \in (c_f, r_0)$, similarly as in Case IV one can verify that (1) during peak periods, all committed full-timers and available part-timers will participate, (2) during off-peak periods still all committed full-timers will participate but no part-timers will do so, and (3) full-timers will commit with a mixed strategy since the expected utility on the platform exactly equals their outside option. Hence, the company's optimization problem is thus to control full-timers' commitment probability $q$ and the earning rate during peak periods $r_{\mathsf{h}}$, i.e.,

$$\begin{aligned} \max_{r_{\mathsf{h}} \in (c_a, \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}), q \in [0,1]} \quad & \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{h}}) \\ + \quad & \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}}}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f. \end{aligned}$$

As $c_a$ increases, one can see that the feasibility space (i.e., $(c_a, \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}) \times [0,1]$) shrinks. This implies that the company's optimal profit $\Pi^*$ will decrease in $c_a$.

*Case V:* $r_h = (r_0 - \beta_l c_f)/\beta_h, r_l = c_f$. Similarly as in Case I, one can verify that $\Pi^*$ in this case will be constant in $c_a$.

In summary, we have verified that the function value in any Case from I to IV is either increasing, or decreasing or first decreasing and then increasing in $c_a$. As such, the company's optimal profit $\Pi^*$, which is the maximum among all the functions from Case I to IV, must be first decreasing and then increasing in $c_a$ in general.[27] Since the optimal profit $\Pi^*_{fo}$ in the full-timer-only CM is constant of $c_a$, there shall exist $\underline{c}'', \overline{c}''$ such that $\Pi^* \le \Pi^*_{fo}$ if $c_a \in [\underline{c}'', \overline{c}'']$.[28]

**Proof of Proposition 2**

We first verify Proposition 2(ii). Note that,

$$\frac{\partial^2 \Pi_E}{\partial q \partial B} = -(\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \frac{\beta_t \mu_t}{(\mu_t - q\lambda_f)^2} + \frac{1}{\tau})\lambda_f < 0,$$

which implies that the optimal fraction of full-timers to hire $q^*_E$ and thus the equilibrium transaction volume $\lambda^*_E = q^*_E \lambda_f$ are decreasing in $B$. Then for part(iii), using the envelop theorem, we have $\partial \Pi_E / \partial B = -(\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t/(\mu_t - q^*_E\lambda_f) + \frac{1}{\tau})q^*_E\lambda_f < 0$, which implies that the company's optimal profit $\Pi^*_E$ is also decreasing in $B$.

We now verify part(i). We essentially attempt to verify that $\partial S^*_E/\partial B$ will first be positive and then negative as $B$ increases. To this end, note that the first-order condition with respect to $\lambda^*_E$ is

$$\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t p_t - r_B(\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \frac{\beta_t \mu_t}{(\mu_t - \lambda^*_E)^2} + \frac{1}{\tau}) = 0 \Rightarrow B = \frac{\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t p_t}{\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \frac{\beta_t \mu_t}{(\mu_t - \lambda^*_E)^2} + \frac{1}{\tau}} - r_0.$$

As such, the analysis of $\partial S^*_E/\partial B$ can be transformed to that of $\partial S^*_E/\partial \lambda^*_E$. For simplicity let $y \equiv \lambda^*_E$. Further define $\nu_p = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t p_t$. After some algebraic manipulations, we have

$$\frac{\partial S^*_E}{\partial y} = \nu_p \frac{3 - 2\frac{\frac{1}{\tau} + \frac{\beta_\mathsf{h}\mu_\mathsf{h}}{(\mu_\mathsf{h}-y)^2}\frac{\mu_\mathsf{h}}{\mu_\mathsf{h}-y} + \frac{\beta_\mathsf{l}\mu_\mathsf{l}}{(\mu_\mathsf{l}-y)^2}\frac{\mu_\mathsf{l}}{\mu_\mathsf{l}-y}}{\frac{1}{\tau} + \frac{\beta_\mathsf{h}\mu_\mathsf{h}}{(\mu_\mathsf{h}-y)^2} + \frac{\beta_\mathsf{l}\mu_\mathsf{l}}{(\mu_\mathsf{l}-y)^2}}}{\frac{1}{\tau} + \frac{\beta_\mathsf{h}\mu_\mathsf{h}}{(\mu_\mathsf{h}-y)^2} + \frac{\beta_\mathsf{l}\mu_\mathsf{l}}{(\mu_\mathsf{l}-y)^2}} - r_0 \qquad \text{(OA.12)}$$

Given that

$$\frac{\partial S^*_E}{\partial y}|_{y=0} = \frac{\nu_p}{\frac{1}{\tau} + \frac{\beta_\mathsf{h}}{\mu_\mathsf{h}} + \frac{\beta_\mathsf{l}}{\mu_\mathsf{l}}} - r_0 > 0, \ \lim_{y\to\mu_\mathsf{l}} \frac{\partial S^*_E}{\partial y} = -\infty,$$

[27] Since there are only finite number of functions from Case I to IV, there can be only a finite number of interactions between any two of them, and therefore there are only finite number of segments. Take the maximum among all functions on each segment. Suppose on one such segment the maximum value is first decreasing and then increasing (in $c_a$). Then clearly the function value to the left of this segment can only be decreasing (in $c_a$), and to the right of this segment can only be increasing (in $c_a$). Similar arguments can be made should the function value on this particular segment is increasing or decreasing in $c_a$.

[28] For $[\underline{c}'', \overline{c}'']$ to be nonempty, we need extra conditions on other primitives. For conciseness, we omit the discussion here and elsewhere on similar issues.

it suffices to show that $\partial S_E^*/\partial y$ has only one zero point on $(0,\mu_{\mathsf{l}})$. The key is then to show that $\partial S_E^*/\partial y$ is decreasing in $y$. Note that, the denominator in the first component of (OA.12) is increasing in $y$. Hence it is sufficient to verify that the nominator is decreasing in $y$. This is verified as follows,

$$\frac{\partial}{\partial y}\left(\frac{\frac{1}{\tau}+\frac{\beta_{\mathsf{h}}\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}}-y)^2}\frac{\mu_{\mathsf{h}}}{\mu_{\mathsf{h}}-y}+\frac{\beta_{\mathsf{l}}\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}}-y)^2}\frac{\mu_{\mathsf{l}}}{\mu_{\mathsf{l}}-y}}{\frac{1}{\tau}+\frac{\beta_{\mathsf{h}}\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}}-y)^2}+\frac{\beta_{\mathsf{l}}\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}}-y)^2}}\right)$$
$$=\frac{\frac{2y}{\tau}\left(\frac{\beta_{\mathsf{h}}\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}}-y)^4}+\frac{\beta_{\mathsf{l}}\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}}-y)^4}\right)+\left(\frac{\beta_{\mathsf{h}}\mu_{\mathsf{h}}^2}{(\mu_{\mathsf{h}}-y)^4}+\frac{\beta_{\mathsf{l}}\mu_{\mathsf{l}}^2}{(\mu_{\mathsf{l}}-y)^4}\right)\left(\frac{1}{\tau}+\frac{\beta_{\mathsf{h}}\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}}-y)^2}+\frac{\beta_{\mathsf{l}}\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}}-y)^2}\right)+\frac{2\beta_{\mathsf{h}}\beta_{\mathsf{l}}\mu_{\mathsf{h}}\mu_{\mathsf{l}}(\mu_{\mathsf{h}}-\mu_{\mathsf{l}})^2 y}{(\mu_{\mathsf{h}}-y)^4(\mu_{\mathsf{l}}-y)^4}}{\left(\frac{1}{\tau}+\frac{\beta_{\mathsf{h}}\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}}-y)^2}+\frac{\beta_{\mathsf{l}}\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}}-y)^2}\right)^2}\geq 0.\square$$

**Proof of Theorem 1**

The results follow by combining Propositions 1 and 2. $\square$

**Proof of Theorem 2**
**Proof of Theorem 2 Part I**

Define $\overline{B}_+=\tau p_{\mathsf{h}}-r_0$. For $B\geq\overline{B}_+$, the wage floor $r_B/\tau$ is higher than even the service price in peak periods (i.e., $r_B/\tau\geq p_{\mathsf{h}}$). The company then shuts down and full-timers' welfare in the C$^+$M equilibrium $S_+^*=0\leq S^*$.

Next define $\underline{B}_+=\tau p_{\mathsf{l}}-r_0$. For $B\leq\underline{B}_+$, the wage floor $r_B/\tau$ is lower than service prices in both peak and in off-peak periods (i.e., $r_B/\tau\leq p_t$ for $t\in\{\mathsf{h},\mathsf{l}\}$), and the company will keep operating throughout the day. We now verify that for $B\leq\underline{B}_+$, the full-timers' welfare in the C$^+$M is (weakly) higher than in the CM (i.e., $S_+^*\geq S^*$). If the wage floor $r_B/\tau$ in the C$^+$M indeed raises the optimal piece-rate wage in any period (i.e., $w_{+,t}^*\geq w_t^*$ for $t\in\{\mathsf{h},\mathsf{l}\}$), then according to the Proof of Lemma 1, the average earning rates both in peak and off-peak periods and thus full-timers' welfare will be higher in the C$^+$M than in the CM (i.e., $S_+^*\geq S^*$). Chances are that the optimal piece-rate wages in the CM may be sufficiently low in some periods but sufficiently high in the others; as the wage floor $r_B/\tau$ in the C$^+$M pushes up the low wages in some periods, the company may no longer find it optimal to pay the same high piece-rate wages in the other periods as in the CM (think about the *temporal incentive pooling* strategy we analyze in Section 3.1). As a result, the wage floor $r_B/\tau$ may result in higher wages in the C$^+$M than in the CM in some periods but lower wages in the others (i.e., $w_{+,t}^*\geq w_t^*$ but $w_{+,-t}^*<w_{-t}^*$ for some $t\in\{\mathsf{h},\mathsf{l}\}$) Below we will show that even in such cases we still have $S_+^*\geq S^*$.

Lemma. *We have $S_+^*\geq S^*$ even when $w_{+,t}^*\geq w_t^*$ but $w_{+,-t}^*<w_{-t}^*$ for some $t\in\{h,l\}$.*

Proof. Suppose the opposite is true, i.e., $S^* > S^*_+$. This implies that $S^* > 0$, which in turn implies that $c_a > r_0$ according to Lemma OA.10. To streamline the discussion, we focus on the case with $c_a \in (r_0, (r_0 - \beta_\mathsf{h} c_f)/\beta_\mathsf{l})$. The underlying rationale applies to the other cases.

As we show in the Proof of Proposition 1, when $c_a \in (r_0, (r_0 - \beta_\mathsf{h} c_f)/\beta_\mathsf{l})$, for full-timers' welfare to be strictly positive in the CM equilibrium (i.e., $S^* > 0$), the optimal piece-rate wages must be $w^*_t = c_a(1/(\mu_t - \lambda_f - q^*_{a,t}\lambda_{a,t})$ for $t \in \{\mathsf{h}, \mathsf{l}\}$, where $q^*_{a,t}$ are the optimal solutions to the following concave optimization program,

$$\max_{(q_{a,\mathsf{h}}, q_{a,\mathsf{l}}) \in [0,1]^2} \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t (p_t - c_a(\frac{1}{\mu_t - \lambda_f - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}))(\lambda_f + q_{a,t}\lambda_{a,t}).$$

Now suppose the wage floor $r_B/\tau$ in the C$^+$M is lower than $w^*_\mathsf{h}$ but higher than $w^*_\mathsf{l}$; that is, the wage floor will only enforce in off-peak periods (i.e., $w^*_{+,\mathsf{l}} \geq r_B/\tau \geq w^*_\mathsf{l}$). But then there is no need for the company to strictly lower the wage in peak periods, since (a) $w_\mathsf{h} = w^*_\mathsf{h} \geq r_B/\tau$ is still feasible in the C$^+$M and (b) the concavity of the company's profit ensures that any $w > w^*_\mathsf{h}$ will be suboptimal. Therefore, we must have $w^*_{+,\mathsf{h}} = w^*_\mathsf{h}$, but this (together with $w^*_{+,\mathsf{l}} \geq w^*_\mathsf{l}$) contradicts that $w^*_{+,t} \geq w^*_t$ but $w^*_{+,-t} < w^*_{-t}$ for some $t \in \{\mathsf{h}, \mathsf{l}\}$. Such a contradiction also exists when the wage floor $r_B/\tau$ in the C$^+$M is lower than $w^*_\mathsf{l}$ but higher than $w^*_\mathsf{h}$. □

**Proof of Theorem 2 Part II**

We now verify Theorem 2 parts(i), (ii) and (iii) in order.

For Part(i), if $B \leq \min_{t\in\{\mathsf{h},\mathsf{l}\}} \tau w^*_t - r_0$, the wage floor $r_B/\tau$ in the C$^+$M will be lower than $\min_{t\in\{\mathsf{h},\mathsf{l}\}} w^*_t$; that is, the wage floor will not enforce. Thus, the C$^+$M boils down to the CM.

For Part(ii), we define $\bar{M}_+ \equiv \mu_\mathsf{h}/\tau$. That $M_f \geq \bar{M}_+$ implies that full-timers' gross arrival rate $\lambda_f = M_f\tau \geq \mu_\mathsf{h} > \mu_\mathsf{l}$. Then the equilibrium commitment probability among full-timers $q^*_+$ must be strictly less than 1 in the C$^+$M. To see this, suppose instead $q^*_+ = 1$. Lemma OA.7 implies that there must be a $t \in \{\mathsf{h}, \mathsf{l}\}$ such that committed full-timers' participation probability $q_{f,t} = 1$. But then the transaction volume in that period $\lambda_t \geq \lambda_f \geq \mu_\mathsf{h}$, and this will render the average wait time on the platform prohibitively long, which in turn drives the average earning rate $r_t$ down to zero and thus reduces $q_{f,t}$ to 0 as well. Contradiction. As such, we shall always have $q^*_+ < 1$ and, according to Lemma OA.8, full-timers' surplus $S^*_+$ equals 0 in the C$^+$M equilibrium.

We now verify Part(iii). The idea for the proof is that, we will analyze how the equilibrium average earnings rates $r^*_{+,\mathsf{h}}$ and $r^*_{+,\mathsf{l}}$ in the C$^+$M is determined by the wage floor $r_B/\tau$, and then show that both $r^*_{+,\mathsf{h}}$ and $r^*_{+,\mathsf{l}}$—and thus the full-timers' expected utility $u^*_+$ in equilibrium—are decreasing in the labor pool size of part-timers $M_a$. Note that, Lemma OA.10 implies that to have

$S^* > 0$ we must have $c_a > r_0$. For succinctness, below we elaborate the idea with the case where $c_a \in (r_0, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$. The remaining scenarios (i.e., $c_a > (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$) can be verified similarly.

In the Proof of Proposition 1(i), we show that when $c_a \in (r_0, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$, if full-timers' surplus $S^* > 0$ in the CM equilibrium, the company must have set the optimal earning rates as $r^*_{\mathsf{h}} = r^*_{\mathsf{l}} = c_a$. As such, all full-timers will commit to the platform and participate all day long, and the company's profit maximization problem is to control part-timers' participation probabilities $q_{a,\mathsf{h}}$ and $q_{a,\mathsf{l}}$; see the Lemma in the Proof of Proposition 1(i) for more details. Denote the optimal part-timers' participation probabilities in the CM by $q^*_{a,\mathsf{h}}$ and $q^*_{a,\mathsf{l}}$. Then according to Lemma OA.2, the optimal piece-rate wages in the CM are essentially

$$w^*_t = c_a\Big(\frac{1}{\mu_t - \lambda_f - q^*_{a,t}\lambda_{a,t}} + \frac{1}{\tau}\Big), t \in \{\mathsf{h}, \mathsf{l}\}.$$

Now, suppose $B$ is sufficiently low such that $r_B/\tau \le \min_{t\in\{\mathsf{h},\mathsf{l}\}} w^*_t$. Then clearly the optimal piece-rate wages in the C$^+$M are the same as in the CM, and so are the corresponding average earning rates, i.e., $r^*_{+,\mathsf{h}} = r^*_{+,\mathsf{l}} = c_a$. Then suppose $r_B/\tau > w^*_{\mathsf{h}}$ while $r_B/\tau \le w^*_{\mathsf{l}}$, which imply that the off-peak wage $w^*_{\mathsf{l}}$ in the CM is still feasible (and thus still optimal) in the C$^+$M while the peak wage $w^*_{\mathsf{h}}$ is no longer so. To pin down the optimal piece-rate wage during peak periods for the C$^+$M, notice that the company's profit function (OA.2) is concave in part-timers' participation probability $q_{a,\mathsf{h}}$. This implies that the constraint $w_{\mathsf{h}} \ge r_B/\tau$ will push up part-timers' participation probability $q_{a,\mathsf{h}}$ until it reaches 1; that is, the optimal peak wage $w^*_{+,\mathsf{h}}$ will equal $r_B/\tau$, the corresponding average earning rate $r^*_{+,\mathsf{h}}$ will still be $c_a$, and the optimal participation probability $q_{a,\mathsf{h}}$ be such that $w^*_{+,\mathsf{h}} = r_B/\tau$. If $q_{a,\mathsf{h}}$ has already reach 1, then all available workers, full-time or part-time, are attracted to the platform. Hence the corresponding average earning rate $r^*_{+,\mathsf{h}} = r_B/\tau/(1/(\mu_{\mathsf{h}} - \lambda_f - \lambda_{a,t}) + 1/\tau)$, which will be higher than $c_a$. In summary, we have

$$r^*_{+,\mathsf{h}} = \begin{cases} c_a & \text{if } r_B \le \tau \cdot c_a\big(\frac{1}{\mu_{\mathsf{h}} - \lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}\big), \\ \frac{r_B/\tau}{\frac{1}{\mu_{\mathsf{h}} - \lambda_f - \lambda_{a,t}} + \frac{1}{\tau}} & \text{otherwise,} \end{cases}$$

which can be verified to decrease in part-timers' gross arrival rate $\lambda_{a,t}$ and thus their labor pool size $M_a$.

For the remaining cases (i.e., the one with $r_B/\tau \le w^*_{\mathsf{h}}$ but $r_B/\tau > w^*_{\mathsf{l}}$ and the one with $r_B/\tau > w^*_t$ for $t \in \{\mathsf{h}, \mathsf{l}\}$), one can similarly verify that the optimal average earning rates $r^*_{+,\mathsf{h}}$ and $r^*_{+,\mathsf{l}}$ in the C$^+$M will decrease in $M_a$. Therefore, full-timers' expected utility $u^*_+$ in the C$^+$M equilibrium must be decreasing in $M_a$ as well. We thus have

$$\frac{S^*_+ - S^*}{S^*} = \frac{u^*_+}{u^*} - 1 = \frac{u^*_+}{c_a - c_f} - 1$$

decreasing in $M_a$. $\square$

**Proof of Theorem 3**

The result for EM versus CM follows Proposition 2(ii) and that the transaction volume $\lambda^*$ in the CM is constant of $B$.

For C$^+$M versus CM, See Proof of Theorem 2 Part I for the existence of $\underline{B}_+$. Define $\overline{B}'_+ \equiv \max_{t\in\{\mathsf{h},\mathsf{l}\}} \tau w_t^* - r_0$, where $w_t^*$ denotes the optimal piece-rate wage in the CM. For $B \geq \overline{B}'_+$, the wage floor $r_B/\tau$ in the C$^+$M will be higher than the optimal wages in the CM both in peak and in off-peak periods. Hence the optimal wages in the C$^+$M must be higher than those in the CM both in peak and in off-peak periods, i.e., $w_{+,t}^* \geq w_t^*$ for $t \in \{\mathsf{h},\mathsf{l}\}$. Then according to the monotonicity of the transaction volume which we establish in Corollary OA.6, we have $\lambda_+^* \geq \lambda^*$. $\square$

**Proof of Theorem 4**
**Proof of Theorem 4 Part I**

The inequality $\Pi_+^* \leq \Pi^*$ immediately follows the fact that the wage floor $r_B/\tau$ in the C+M shrinks the set of feasible piece-rate wages for the company.

**Proof of Theorem 4 Part II**

We next verify the result that the company's optimal profit is lower in the EM than in the CM, i.e., $\Pi_E^* \leq \Pi^*$. To this end, recall that $\lambda_E^*$ denotes the optimal workforce in the EM, and below is the company's optimal profit in the EM

$$\begin{aligned}\Pi_E^* &= \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_B(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_B(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* \\ &\leq \bar{\Pi}_E^* \equiv \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^*, \qquad \text{(OA.13)}\end{aligned}$$

and the inequality is given by that $\Pi_E^*$ is decreasing in the lump-sum benefit $B$, which we show in Proposition 2. Also recall that we use $\lambda_{a,\mathsf{h}}$ and $\lambda_{a,\mathsf{l}}$ to denote part-timers' *arrival* rates and $\lambda_{\mathsf{h}}$ and $\lambda_{\mathsf{l}}$ to denote the *participation* rates of full- and part-timers combined during peak and off-peak periods in the CM, respectively. Finally, recall the presumption that for full-timers, their reservation earning rate $r_0$ is strictly higher than their average time value $c_f$, i.e., $r_0 > c_f$, and note that for simplicity we assume $c_a \neq r_0$ and $c_a \neq c_f$.

The key to the proof is properly lower bounding the company's optimal profit $\Pi^*$ in the CM—which can be done by specifying a feasible earning rate schedule $(r_{\mathsf{h}}, r_{\mathsf{l}})$ and a worker participation rate schedule $(\lambda_{\mathsf{h}}, \lambda_{\mathsf{l}})$—and then show that such a lower bound is already higher than $\Pi_E^*$. To proceed, consider the following cases.

*Case I:* $c_a > r_0 > c_f$. In this case, it is essentially more costly for the company to incentivize part-timers than full-timers. One feasible pair of piece-rate wages for the company $(w_{\mathsf{h}}, w_{\mathsf{l}})$ in the

CM is such that the induced earning rates in both peak and off-peak periods $r_{\mathsf{h}}$ and $r_{\mathsf{l}}$ equal to full-timers' reservation earning rate $r_0$. Then no part-timers will ever participate since the average earning rate in either period is strictly less than their time value, i.e., $r_t < c_a$ for $t \in \{\mathsf{h}, \mathsf{l}\}$, whereas committed full-timers will participate in both periods since $r_{\mathsf{h}} = r_{\mathsf{l}} > c_f$. As such, the company can set the committing probability among full-timers $q$ to be exactly the optimal fraction of full-timers to hire in the EM $q_E^*$, and workers' participation rates in the CM will thus be $\lambda_{\mathsf{h}} = \lambda_{\mathsf{l}} = \lambda_E^*$. Then by construction the feasible piece-rate wages $(w_{\mathsf{h}}, w_{\mathsf{l}})$ are

$$w_{\mathsf{h}} = r_0\left(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}\right), w_{\mathsf{l}} = r_0\left(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}\right).$$

Plugging $(w_{\mathsf{h}}, w_{\mathsf{l}})$ in the company's profit function $\Pi$, we obtain a lower bound for the company's optimal profit $\Pi^*$ in the CM and we can then compare $\Pi^*$ and $\Pi_E^*$ as follows.

$$\Pi^* \geq \beta_{\mathsf{h}}\left(p_{\mathsf{h}} - r_0\left(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}\right)\right)\lambda_E^* + \beta_{\mathsf{l}}\left(p_{\mathsf{l}} - r_0\left(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}\right)\right)\lambda_E^* \geq \Pi_E^*.$$

*Case II: $c_a \in (c_f, r_0)$.* Compared with Case I, it essentially becomes less costly for the company to incentivize part-timers. We further break down the analysis into four subcases.

*Case II.A: $\lambda_{a,t} \geq \lambda_E^*$ for any $t \in \{h, l\}$.* A feasible solution for the company in the CM is to set $r_{\mathsf{h}} = r_{\mathsf{l}} = c_a$. As such, only part-timers will ever join the platform because for the full-timers the expected utility of working on the platform $u = \beta_{\mathsf{h}}(r_{\mathsf{h}} - c_f)^+ + \beta_{\mathsf{l}}(r_{\mathsf{l}} - c_f)^+ = c_a - c_f < u_0 = r_0 - c_f$. Then the company's problem is to optimally control part-timers' joining rates in each period. In particular, the company can set part-timers' participation probability during peak periods $q_{a,h} = \lambda_E^*/\lambda_{a,h}$ and similarly that during off-peak periods as $q_{a,l} = \lambda_E^*/\lambda_{a,\mathsf{l}}$. We thus have $\lambda_t = q_{a,t}\lambda_{a,t} = \lambda_E^*$ for $t \in \{\mathsf{h}, \mathsf{l}\}$ and

$$\begin{aligned}\Pi^* &\geq \beta_{\mathsf{h}}\left(p_{\mathsf{h}} - c_a\left(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}\right)\right)\lambda_E^* + \beta_{\mathsf{l}}\left(p_{\mathsf{l}} - c_a\left(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}\right)\right)\lambda_E^* \\ &\geq \beta_{\mathsf{h}}\left(p_{\mathsf{h}} - r_0\left(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}\right)\right)\lambda_E^* + \beta_{\mathsf{l}}\left(p_{\mathsf{l}} - r_0\left(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}\right)\right)\lambda_E^* \geq \Pi_E^*\end{aligned}$$

*Case II.B: $\lambda_{a,h} < \lambda_E^*$ and $\lambda_{a,l} \geq \lambda_E^*$.* A feasible solution for the company in the CM is to set $r_{\mathsf{h}} = (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}} > c_a$ and $r_{\mathsf{l}} = c_a$. Then for full-timers, the utility of working on the platform $u = \beta_{\mathsf{h}}(r_{\mathsf{h}} - c_f)^+ + \beta_{\mathsf{l}}(r_{\mathsf{l}} - c_f)^+ = u_0$, which implies that in stage 0 there will be a mixed-strategy equilibrium among full-timers, i.e., each full-timer will choose to commit with some probability $q \in [0,1]$. Note that, because $r_{\mathsf{h}} > c_f$ and $r_{\mathsf{l}} > c_f$, those committed full-timers will all participate in both peak and off-peak periods; in other words, the effective worker participation rate is at least $q\lambda_f$ in both periods.

Now, because $r_{\mathsf{l}} = c_a$, during off-peak periods there will be a mixed strategy equilibrium among part-timers, i.e., each part-timer participates with some probability $q_{a,\mathsf{l}} \in [0,1]$. In contrast, as $r_{\mathsf{h}} = (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}} > c_a$, during peak-periods all part-timers would love to participate. As such, the service level during peak periods is $\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}}$, and that during off-peak periods is $\lambda_{\mathsf{l}} = q\lambda_f + q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}$. Given the conditions $\lambda_{a,\mathsf{h}} < \lambda_E^*$ and $\lambda_{a,\mathsf{l}} \geq \lambda_E^*$, the company can find such $q$ and $q_{a,\mathsf{l}}$ that enforce $\lambda_{\mathsf{h}} = \lambda_{\mathsf{l}} = \lambda_E^*$. Then we have

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - \frac{r_0 - \beta_{\mathsf{l}} c_a}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^*.$$

Then combined with the inequality (OA.13),

$$\begin{aligned}\Pi^* - \Pi_E^* &\geq -\beta_{\mathsf{h}}(\frac{r_0 - \beta_{\mathsf{l}} c_a}{\beta_{\mathsf{h}}} - r_0)\frac{\lambda_E^*}{\mu_{\mathsf{h}} - \lambda_E^*} + \beta_{\mathsf{l}}(r_0 - c_a)\frac{\lambda_E^*}{\mu_{\mathsf{l}} - \lambda_E^*}\\ &\geq \beta_{\mathsf{l}}(r_0 - c_a)(\frac{\lambda_E^*}{\mu_{\mathsf{l}} - \lambda_E^*} - \frac{\lambda_E^*}{\mu_{\mathsf{h}} - \lambda_E^*}) \geq 0.\end{aligned}$$

The last inequality holds because $\mu_{\mathsf{h}} \geq \mu_{\mathsf{l}}$.

*Case II.C:* $\lambda_{a,h} > \lambda_E^*$ *and* $\lambda_{a,l} \leq \lambda_E^*$. Recall $\lambda_{E,t}^\dagger, t \in \{\mathsf{h}, \mathsf{l}\}$ we have defined in Lemma OA.11. If $\lambda_{a,\mathsf{l}} \geq \lambda_{E,\mathsf{l}}^\dagger$, the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = c_a$ and then make $q_{a,\mathsf{h}} = \lambda_E^*/\lambda_{a,\mathsf{h}}$ and $q_{a,\mathsf{l}} = \lambda_{E,\mathsf{l}}^\dagger/\lambda_{a,\mathsf{l}}$. Then the company's profit

$$\begin{aligned}\Pi^* &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda_{E,\mathsf{l}}^\dagger} + \frac{1}{\tau}))\lambda_{E,\mathsf{l}}^\dagger\\ &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* \geq \Pi_E^*.\end{aligned}$$

If instead $\lambda_{a,\mathsf{l}} < \lambda_{E,\mathsf{l}}^\dagger$, suppose first $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \leq \lambda_{E,\mathsf{h}}^\dagger - \lambda_{E,\mathsf{l}}^\dagger$. The company can still set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and choose a committing probability $q$ among full-timers such that $\lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}} = \lambda_{E,\mathsf{l}}^\dagger$; then naturally we have $\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}} \in (\lambda_E^*, \lambda_{E,\mathsf{h}}^\dagger]$. As such,

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_{E,\mathsf{l}}^\dagger} + \frac{1}{\tau}))\lambda_{E,\mathsf{l}}^\dagger \geq \Pi_E^*,$$

where the last inequality is essentially given by Lemma OA.11.

Then suppose $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \geq \lambda_{E,\mathsf{h}}^\dagger - \lambda_{E,\mathsf{l}}^\dagger$. The analysis for this scenario follows essentially the same idea as in the upcoming Case II.D. We thus omit the discussion here.

*Case II.D:* $\lambda_{a,t} \leq \lambda_E^*$ *for any* $t \in \{h, l\}$. Recall $\lambda_{E,\mathsf{h}}^\dagger$ and $\lambda_{E,\mathsf{l}}^\dagger$ that we have defined previously.

First, suppose $\lambda_{a,\mathsf{h}} > \lambda_{a,\mathsf{l}}$ and $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \leq \lambda_{E,\mathsf{h}}^\dagger - \lambda_{E,\mathsf{l}}^\dagger$. Then the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and make sure full-timers' committing probability $q$ satisfy

$$\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,h} \in [\lambda_E^*, \lambda_{E,\mathsf{h}}^\dagger],\ \lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,l} \in [\lambda_{E,\mathsf{l}}^\dagger, \lambda_E^*].$$

Then we have

$$\begin{aligned}\Pi^* &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_{\mathsf{l}}} + \frac{1}{\tau}))\lambda_{\mathsf{l}} \\ &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* \geq \Pi_E^*,\end{aligned}$$

of which the second inequality is given by the definitions of $\lambda_{E,\mathsf{h}}^\dagger$ and $\lambda_{E,\mathsf{l}}^\dagger$ as well as the concavity of company's profit $\Pi_E$ in workforce $\lambda$; see the proof of Lemma OA.11 for more details.

Next, we consider the scenario with $\lambda_{a,\mathsf{h}} > \lambda_{a,\mathsf{l}}$ but $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \geq \lambda_{E,\mathsf{h}}^\dagger - \lambda_{E,\mathsf{l}}^\dagger$. Denote the term $\mu_t - \sqrt{\mu_t \tau r_0/(p_t\tau - r_0)}$ defined in Lemma OA.11 as $\bar{\lambda}_{E,t}$. We shall examine the following sub-scenarios.

(a) $\lambda_f \leq \bar{\lambda}_{E,\mathsf{l}} < \bar{\lambda}_{E,\mathsf{h}}$. According to Lemma OA.11, we have $\lambda_E^* = \lambda_{E,\mathsf{h}}^\dagger = \lambda_{E,\mathsf{l}}^\dagger = \lambda_f$. If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \leq \bar{\lambda}_{E,\mathsf{h}} - \lambda_f$, the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and choose a committing probability $q$ among full-timers such that

$$\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}} \in (\lambda_f, \bar{\lambda}_{E,\mathsf{h}}], \lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}} = \lambda_f.$$

As such, we have

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_f} + \frac{1}{\tau}))\lambda_f \geq \Pi_E^*.$$

If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} > \bar{\lambda}_{E,\mathsf{h}} - \lambda_f$, the company can set $r_{\mathsf{h}} = c_a, r_{\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} c_a)/\beta_{\mathsf{l}}$ and choose a committing probability $q$ among full-timers and a participation probability $q_{a,\mathsf{h}}$ among part-timers during peak periods such that

$$q\lambda_f + q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}} = \bar{\lambda}_{E,\mathsf{h}}, q\lambda_f + \lambda_{a,\mathsf{l}} = \lambda_f.$$

We then have

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} + \frac{1}{\tau}))\bar{\lambda}_{E,\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - \lambda_f} + \frac{1}{\tau}))\lambda_f$$

and, combining the inequality (OA.13),

$$\Pi^* - \Pi_E^* \geq \beta_{\mathsf{h}}(r_0 - c_a)(\frac{1}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} + \frac{1}{\tau})\bar{\lambda}_{E,\mathsf{h}} - \beta_{\mathsf{h}}(r_0 - c_a)(\frac{1}{\mu_{\mathsf{l}} - \lambda_f} + \frac{1}{\tau})\lambda_f \geq 0.$$

The last inequality holds because $\bar{\lambda}_{E,\mathsf{h}} > \bar{\lambda}_{E,\mathsf{l}} \geq \lambda_f$ and

$$\begin{aligned}\frac{\bar{\lambda}_{E,\mathsf{h}}}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} - \frac{\lambda_f}{\mu_{\mathsf{l}} - \lambda_f} &\geq \frac{\bar{\lambda}_{E,\mathsf{h}}}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} - \frac{\bar{\lambda}_{E,\mathsf{l}}}{\mu_{\mathsf{l}} - \bar{\lambda}_{E,\mathsf{l}}} \\ &= \frac{\sqrt{\mu_{\mathsf{h}}(p_{\mathsf{h}}\tau - r_0)}}{\sqrt{\tau r_0}} - \frac{\sqrt{\mu_{\mathsf{l}}(p_{\mathsf{l}}\tau - r_0)}}{\sqrt{\tau r_0}} \geq 0.\end{aligned}$$

(b) $\bar{\lambda}_{E,\mathsf{l}} < \lambda_f < \bar{\lambda}_{E,\mathsf{h}}$. By definition we have $\lambda^{\dagger}_{E,\mathsf{h}} = \lambda_f$ and $\lambda^{\dagger}_{E,\mathsf{l}} = \bar{\lambda}_{E,\mathsf{l}}$. If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \le \bar{\lambda}_{E,\mathsf{h}} - \bar{\lambda}_{E,\mathsf{l}}$, the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and choose a committing probability $q$ among full-timers such that

$$\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}} \in [\lambda_f, \bar{\lambda}_{E,\mathsf{h}}], \lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}} = \bar{\lambda}_{E,\mathsf{l}}.$$

As such, we have

$$\Pi^* \ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \bar{\lambda}_{E,\mathsf{l}}} + \frac{1}{\tau}))\bar{\lambda}_{E,\mathsf{l}} \ge \Pi^*_E.$$

If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} > \bar{\lambda}_{E,\mathsf{h}} - \bar{\lambda}_{E,\mathsf{l}}$, the company can set $r_{\mathsf{h}} = c_a, r_{\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} c_a)/\beta_{\mathsf{l}}$ and choose a committing probability $q$ among full-timers and a participation probability $q_{a,\mathsf{h}}$ among part-timers during peak periods such that

$$q\lambda_f + q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}} = \bar{\lambda}_{E,\mathsf{h}}, q\lambda_f + \lambda_{a,\mathsf{l}} = \bar{\lambda}_{E,\mathsf{l}}.$$

We then have

$$\Pi^* \ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} + \frac{1}{\tau}))\bar{\lambda}_{E,\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - \bar{\lambda}_{E,\mathsf{l}}} + \frac{1}{\tau}))\bar{\lambda}_{E,\mathsf{l}}$$

and thus, similarly as in the sub-scenario (a),

$$\Pi^* - \Pi^*_E \ge \beta_{\mathsf{h}}(r_0 - c_a)(\frac{1}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} + \frac{1}{\tau})\bar{\lambda}_{E,\mathsf{h}} - \beta_{\mathsf{h}}(r_0 - c_a)(\frac{1}{\mu_{\mathsf{l}} - \bar{\lambda}_{E,\mathsf{l}}} + \frac{1}{\tau})\bar{\lambda}_{E,\mathsf{l}} \ge 0.$$

(c) $\bar{\lambda}_{E,\mathsf{l}} < \bar{\lambda}_{E,\mathsf{h}} \le \lambda_f$. By definition we have $\lambda^{\dagger}_{E,\mathsf{h}} = \bar{\lambda}_{E,\mathsf{h}}$ and $\lambda^{\dagger}_{E,\mathsf{l}} = \bar{\lambda}_{E,\mathsf{l}}$. If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \le \bar{\lambda}_{E,\mathsf{h}} - \bar{\lambda}_{E,\mathsf{l}}$, the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and choose a committing probability $q$ among full-timers such that

$$\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}} \in [\lambda^*_E, \bar{\lambda}_{E,\mathsf{h}}], \lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}} \in [\bar{\lambda}_{E,\mathsf{l}}, \lambda^*_E].$$

We then have

$$\begin{aligned}\Pi^* &\ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_{\mathsf{l}}} + \frac{1}{\tau}))\lambda_{\mathsf{l}} \\ &\ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \lambda^*_E} + \frac{1}{\tau}))\lambda^*_E + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda^*_E} + \frac{1}{\tau}))\lambda^*_E \ge \Pi^*_E,\end{aligned}$$

of which the second inequality is given by the definitions of $\lambda^{\dagger}_{E,\mathsf{h}}$ and $\lambda^{\dagger}_{E,\mathsf{l}}$ as well as the concavity of company's profit $\Pi_E$ in workforce $\lambda$.

If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} > \bar{\lambda}_{E,\mathsf{h}} - \bar{\lambda}_{E,\mathsf{l}}$, then the current sub-scenario is essentially equivalent to the sub-scenario (b). Following the discussion there one can see that again $\Pi^* \ge \Pi^*_E$.

Finally, we discuss the scenario with $\lambda_{a,\mathsf{h}} \le \lambda_{a,\mathsf{l}}$. The company can set $r_{\mathsf{h}} = (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}$, $r_{\mathsf{l}} = c_a$ and find such $q$ and $q_{a,\mathsf{l}}$ that enforce $\lambda_{\mathsf{h}} = \lambda_{\mathsf{l}} = \lambda^*_E$. Then we have

$$\Pi^* \ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - \frac{r_0 - \beta_{\mathsf{l}} c_a}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}} - \lambda^*_E} + \frac{1}{\tau}))\lambda^*_E + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda^*_E} + \frac{1}{\tau}))\lambda^*_E.$$

Then combined with the inequality (OA.13),

$$\begin{aligned}\Pi^* - \Pi_E^* &\geq -\beta_{\mathsf{h}}(\frac{r_0 - \beta_{\mathsf{l}} c_a}{\beta_{\mathsf{h}}} - r_0)\frac{\lambda_E^*}{\mu_{\mathsf{h}} - \lambda_E^*} + \beta_{\mathsf{l}}(r_0 - c_a)\frac{\lambda_E^*}{\mu_{\mathsf{l}} - \lambda_E^*} \\ &\geq \beta_{\mathsf{l}}(r_0 - c_a)(\frac{\lambda_E^*}{\mu_{\mathsf{l}} - \lambda_E^*} - \frac{\lambda_E^*}{\mu_{\mathsf{h}} - \lambda_E^*}) \geq 0.\end{aligned}$$

The last inequality holds because $\mu_{\mathsf{h}} \geq \mu_{\mathsf{l}}$.

*Case III: $c_a < c_f < r_0$.* In this case, it is essentially less costly for the company to incentivize part-timers than full-timers. Similarly as in Case II, here we will proceed with four subcases.

*Case III.A: $\lambda_{a,t} \geq \lambda_E^*$ for any $t \in \{h, l\}$.* One can follow the same analysis as in Case II.A and verify that $\Pi^* \geq \Pi_E^*$.

*Case III.B: $\lambda_{a,h} < \lambda_E^*$ and $\lambda_{a,l} \geq \lambda_E^*$.* The company can set $r_{\mathsf{h}} = (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$, $r_{\mathsf{l}} = c_a$ and choose a committing probability $q$ among full-timers and a participation probability $q_{a,\mathsf{l}}$ among part-timers during off-peak periods such that $q\lambda_f + \lambda_{a,\mathsf{h}} = \lambda_E^*$ and $q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}} = \lambda_E^*$. We then have

$$\begin{aligned}\Pi^* &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^*, \\ \Pi^* - \Pi_E^* &\geq -\beta_{\mathsf{h}}(\frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}} - r_0)\frac{\lambda_E^*}{\mu_{\mathsf{h}} - \lambda_E^*} + \beta_{\mathsf{l}}(r_0 - c_a)\frac{\lambda_E^*}{\mu_{\mathsf{l}} - \lambda_E^*} \\ &\geq \beta_{\mathsf{l}}(r_0 - c_f)(\frac{\lambda_E^*}{\mu_{\mathsf{l}} - \lambda_E^*} - \frac{\lambda_E^*}{\mu_{\mathsf{h}} - \lambda_E^*}) \geq 0.\end{aligned}$$

*Case III.C: $\lambda_{a,h} > \lambda_E^*$ and $\lambda_{a,l} \leq \lambda_E^*$.* Recall $\lambda_{E,\mathsf{h}}^\dagger$ and $\lambda_{E,\mathsf{l}}^\dagger$ we defined previously in Case II. If $\lambda_{a,\mathsf{l}} \geq \lambda_{E,\mathsf{l}}^\dagger$, the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = c_a$ and then make $q_{a,\mathsf{h}} = \lambda_E^*/\lambda_{a,\mathsf{h}}$ and $q_{a,\mathsf{l}} = \lambda_{E,\mathsf{l}}^\dagger/\lambda_{a,\mathsf{l}}$. Then the company's profit

$$\begin{aligned}\Pi^* &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda_{E,\mathsf{l}}^\dagger} + \frac{1}{\tau}))\lambda_{E,\mathsf{l}}^\dagger \\ &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* \geq \Pi_E^*.\end{aligned}$$

If instead $\lambda_{a,\mathsf{l}} < \lambda_{E,\mathsf{l}}^\dagger$, suppose first $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \leq \lambda_{E,\mathsf{h}}^\dagger - \lambda_{E,\mathsf{l}}^\dagger$. The company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and choose a committing probability $q$ among full-timers such that $\lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}} = \lambda_{E,\mathsf{l}}^\dagger$; then naturally we have $\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}} \in (\lambda_E^*, \lambda_{E,\mathsf{h}}^\dagger]$. As such,

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_{E,\mathsf{l}}^\dagger} + \frac{1}{\tau}))\lambda_{E,\mathsf{l}}^\dagger \geq \Pi_E^*.$$

Then suppose instead $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \geq \lambda_{E,\mathsf{h}}^\dagger - \lambda_{E,\mathsf{l}}^\dagger$. The analysis for this scenario follows essentially the same idea as in the upcoming Case III.D. We thus omit the discussion here.

*Case III.D: $\lambda_{a,t} \leq \lambda_E^*$ for any $t \in \{h, l\}$.* First, suppose $\lambda_{a,\mathsf{h}} > \lambda_{a,\mathsf{l}}$ and $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \leq \lambda_{E,\mathsf{h}}^\dagger - \lambda_{E,\mathsf{l}}^\dagger$. Then the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and make sure full-timers' committing probability $q$ satisfy

$$\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,h} \in [\lambda_E^*, \lambda_{E,\mathsf{h}}^\dagger],\ \lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,l} \in [\lambda_{E,\mathsf{l}}^\dagger, \lambda_E^*].$$

Then by the concavity of company's profit in $\lambda$, we have

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - \lambda_{\mathsf{l}}} + \frac{1}{\tau}))\lambda_{\mathsf{l}} \geq \Pi_E^*.$$

Next, we consider the scenario with $\lambda_{a,\mathsf{h}} > \lambda_{a,\mathsf{l}}$ but $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \geq \lambda_{E,\mathsf{h}}^{\dagger} - \lambda_{E,\mathsf{l}}^{\dagger}$. Recall that we denote the term $\mu_t - \sqrt{\mu_t \tau r_0/(p_t\tau - r_0)}$ in Lemma OA.11 as $\bar{\lambda}_{E,t}$. We shall examine the following sub-scenarios.

(a) $\lambda_f \leq \bar{\lambda}_{E,\mathsf{l}} < \bar{\lambda}_{E,\mathsf{h}}$. According to Lemma OA.11, we have $\lambda_{E,\mathsf{h}}^{\dagger} = \lambda_{E,\mathsf{l}}^{\dagger} = \lambda_E^* = \lambda_f$. If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \leq \bar{\lambda}_{E,\mathsf{h}} - \lambda_f$, the company can still set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and choose a committing probability $q$ among full-timers such that

$$\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}} \in (\lambda_f, \bar{\lambda}_{E,\mathsf{h}}], \lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}} = \lambda_f.$$

As such, we have

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \lambda_f} + \frac{1}{\tau}))\lambda_f \geq \Pi_E^*.$$

If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} > \bar{\lambda}_{E,\mathsf{h}} - \lambda_f$, the company can set $r_{\mathsf{h}} = c_f, r_{\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$ and choose a committing probability $q$ among full-timers and a participation probability $q_{f,\mathsf{h}}$ among committed full-timers during peak periods such that

$$q \cdot q_{f,\mathsf{h}}\lambda_f + \lambda_{a,\mathsf{h}} = \bar{\lambda}_{E,\mathsf{h}}, q\lambda_f + \lambda_{a,\mathsf{l}} = \lambda_f.$$

We then have

$$\begin{aligned}
\Pi^* &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_f(\frac{1}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} + \frac{1}{\tau}))\bar{\lambda}_{E,\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - \lambda_f} + \frac{1}{\tau}))\lambda_f, \\
\Pi^* - \Pi_E^* &\geq \beta_{\mathsf{h}}(r_0 - c_f)(\frac{1}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} + \frac{1}{\tau})\bar{\lambda}_{E,\mathsf{h}} - \beta_{\mathsf{h}}(r_0 - c_f)(\frac{1}{\mu_{\mathsf{l}} - \lambda_f} + \frac{1}{\tau})\lambda_f \\
&\geq \beta_{\mathsf{h}}(r_0 - c_f)(\frac{\bar{\lambda}_{E,\mathsf{h}}}{\mu_{\mathsf{h}} - \bar{\lambda}_{E,\mathsf{h}}} - \frac{\bar{\lambda}_{E,\mathsf{l}}}{\mu_{\mathsf{l}} - \bar{\lambda}_{E,\mathsf{l}}}) \geq 0;
\end{aligned}$$

see Case II.D sub-scenario (a) for details.

(b) $\bar{\lambda}_{E,\mathsf{l}} < \lambda_f < \bar{\lambda}_{E,\mathsf{h}}$. By definition we have $\lambda_{E,\mathsf{h}}^{\dagger} = \lambda_f > \lambda_E^* > \lambda_{E,\mathsf{l}}^{\dagger} = \bar{\lambda}_{E,\mathsf{l}}$. If $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \leq \bar{\lambda}_{E,\mathsf{h}} - \bar{\lambda}_{E,\mathsf{l}}$, the company can set $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ and choose a committing probability $q$ among full-timers such that

$$\lambda_{\mathsf{h}} = q\lambda_f + \lambda_{a,\mathsf{h}} \in [\lambda_f, \bar{\lambda}_{E,\mathsf{h}}], \lambda_{\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}} = \bar{\lambda}_{E,\mathsf{l}}.$$

As such, we have

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_0(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}} + \frac{1}{\tau}))\lambda_{\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_0(\frac{1}{\mu_{\mathsf{l}} - \bar{\lambda}_{E,\mathsf{l}}} + \frac{1}{\tau}))\bar{\lambda}_{E,\mathsf{l}} \geq \Pi_E^*.$$

If $\lambda_{a,\mathsf{h}}-\lambda_{a,\mathsf{l}} > \bar\lambda_{E,\mathsf{h}}-\bar\lambda_{E,\mathsf{l}}$, the company can set $r_{\mathsf{h}}=c_f, r_{\mathsf{l}}=(r_0-\beta_{\mathsf{h}}c_f)/\beta_{\mathsf{l}}$ and choose a committing probability $q$ among full-timers and a participation probability $q_{f,\mathsf{h}}$ among committed full-timers during peak periods such that

$$q\cdot q_{f,\mathsf{h}}\lambda_f+\lambda_{a,\mathsf{h}}=\bar\lambda_{E,\mathsf{h}}, q\lambda_f+\lambda_{a,\mathsf{l}}=\bar\lambda_{E,\mathsf{l}}.$$

We then have

$$\begin{aligned}\Pi^* &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}}-c_f(\frac{1}{\mu_{\mathsf{h}}-\bar\lambda_{E,\mathsf{h}}}+\frac{1}{\tau}))\bar\lambda_{E,\mathsf{h}}+\beta_{\mathsf{l}}(p_{\mathsf{l}}-\frac{r_0-\beta_{\mathsf{h}}c_f}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}}-\bar\lambda_{E,\mathsf{l}}}+\frac{1}{\tau}))\bar\lambda_{E,\mathsf{l}},\\ \Pi^*-\Pi_E^* &\geq \beta_{\mathsf{h}}(r_0-c_f)(\frac{1}{\mu_{\mathsf{h}}-\bar\lambda_{E,\mathsf{h}}}+\frac{1}{\tau})\bar\lambda_{E,\mathsf{h}}-\beta_{\mathsf{h}}(r_0-c_f)(\frac{1}{\mu_{\mathsf{l}}-\bar\lambda_{E,\mathsf{l}}}+\frac{1}{\tau})\bar\lambda_{E,\mathsf{l}}\geq 0.\end{aligned}$$

(c) $\bar\lambda_{E,\mathsf{l}}<\bar\lambda_{E,\mathsf{h}}\leq\lambda_f$. By definition we have $\lambda^\dagger_{E,\mathsf{h}}=\bar\lambda_{E,\mathsf{h}}$ and $\lambda^\dagger_{E,\mathsf{l}}=\bar\lambda_{E,\mathsf{l}}$. If $\lambda_{a,\mathsf{h}}-\lambda_{a,\mathsf{l}}\leq\bar\lambda_{E,\mathsf{h}}-\bar\lambda_{E,\mathsf{l}}$, the company can set $r_{\mathsf{h}}=r_{\mathsf{l}}=r_0$ and choose a committing probability $q$ among full-timers such that

$$\lambda_{\mathsf{h}}=q\lambda_f+\lambda_{a,\mathsf{h}}\in[\lambda_E^*,\bar\lambda_{E,\mathsf{h}}], \lambda_{\mathsf{l}}=q\lambda_f+\lambda_{a,\mathsf{l}}\in[\bar\lambda_{E,\mathsf{l}},\lambda_E^*].$$

As such, we have

$$\Pi^* \geq \beta_{\mathsf{h}}(p_{\mathsf{h}}-r_0(\frac{1}{\mu_{\mathsf{h}}-\lambda_{\mathsf{h}}}+\frac{1}{\tau}))\lambda_{\mathsf{h}}+\beta_{\mathsf{l}}(p_{\mathsf{l}}-r_0(\frac{1}{\mu_{\mathsf{l}}-\lambda_{\mathsf{l}}}+\frac{1}{\tau}))\lambda_{\mathsf{l}}\geq\Pi_E^*.$$

If $\lambda_{a,\mathsf{h}}-\lambda_{a,\mathsf{l}}>\bar\lambda_{E,\mathsf{h}}-\bar\lambda_{E,\mathsf{l}}$, then the current sub-scenario is essentially equivalent to the sub-scenario (b). Following the discussion there one can see that again $\Pi^*\geq\Pi_E^*$.

Finally, we discuss the scenario with $\lambda_{a,\mathsf{h}}\leq\lambda_{a,\mathsf{l}}$. The company can set $r_{\mathsf{h}}=(r_0-\beta_{\mathsf{l}}c_f)/\beta_{\mathsf{h}}$, $r_{\mathsf{l}}=c_f$ and find such $q$ and $q_{f,\mathsf{l}}$ that enforce $\lambda_{\mathsf{h}}=\lambda_{\mathsf{l}}=\lambda_E^*$; to be specific,

$$\lambda_{\mathsf{h}}=q\lambda_f+\lambda_{a,\mathsf{h}}=\lambda_E^*, \lambda_{\mathsf{l}}=q\cdot q_{f,\mathsf{l}}\lambda_f+\lambda_{a,\mathsf{l}}=\lambda_E^*.$$

. Then we have

$$\begin{aligned}\Pi^* &\geq \beta_{\mathsf{h}}(p_{\mathsf{h}}-\frac{r_0-\beta_{\mathsf{l}}c_f}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}}-\lambda_E^*}+\frac{1}{\tau}))\lambda_E^*+\beta_{\mathsf{l}}(p_{\mathsf{l}}-c_f(\frac{1}{\mu_{\mathsf{l}}-\lambda_E^*}+\frac{1}{\tau}))\lambda_E^*,\\ \Pi^*-\Pi_E^* &\geq -\beta_{\mathsf{h}}(\frac{r_0-\beta_{\mathsf{l}}c_f}{\beta_{\mathsf{h}}}-r_0)\frac{\lambda_E^*}{\mu_{\mathsf{h}}-\lambda_E^*}+\beta_{\mathsf{l}}(r_0-c_f)\frac{\lambda_E^*}{\mu_{\mathsf{l}}-\lambda_E^*}\\ &\geq \beta_{\mathsf{l}}(r_0-c_f)(\frac{\lambda_E^*}{\mu_{\mathsf{l}}-\lambda_E^*}-\frac{\lambda_E^*}{\mu_{\mathsf{h}}-\lambda_E^*})\geq 0.\end{aligned}$$

**Proof of Theorem 4 Part III**

We now verify the remaining part of Theorem 4; that is, there exist $\underline{B}'$ and $\overline{B}'$ such that if $B\leq\underline{B}'$ or $B\geq\overline{B}'$, the company's profit $\Pi_+^*$ in the C$^+$M equilibrium is higher than that $\Pi_E^*$ in the EM.

Define $\underline{B}' = \min_{t\in\{\mathsf{h},\mathsf{l}\}} \tau w_t^* - r_0$ and $\overline{B}' = \max_{t\in\{\mathsf{h},\mathsf{l}\}} \tau w_t^{II} - r_0$, where $\{w_t^{II}\}$ denotes the feasible wages we construct for the CM in Part II of the proof.

First, for $B \le \underline{B}'$, according to the proof of Theorem 2, the wage floor $r_B/\tau$ in the C$^+$M will not enforce. We thus have $\Pi_+^* = \Pi^* \ge \Pi_E^*$.

Next, for $B \ge \overline{B}'$, consider $w_{+,t} = r_B/\tau$ for both $t \in \{\mathsf{h},\mathsf{l}\}$. Clearly $(w_{+,\mathsf{h}}, w_{+,\mathsf{l}})$ are feasible in the C$^+$M. That $B \ge \overline{B}'$ implies $w_{+,t} \ge w_t^{II}$; according to Corollary OA.6, the transaction volume that can be achieved under wages $(w_{+,\mathsf{h}}, w_{+,\mathsf{l}})$ shall be higher than that under $(w_{\mathsf{h}}^{II}, w_{\mathsf{l}}^{II})$, i.e., $\lambda_{+,t} \ge \lambda_t^{II}$ for any $t \in \{\mathsf{h},\mathsf{l}\}$. Therefore, we have

$$
\begin{aligned}
\Pi_+^* &\ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - w_{+,\mathsf{h}})\lambda_{+,\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - w_{+,\mathsf{l}})\lambda_{+,\mathsf{l}} \\
&\ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - \frac{r_B}{\tau})\lambda_{\mathsf{h}}^{II} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_B}{\tau})\lambda_{\mathsf{l}}^{II} \\
&\ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_B(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}^{II}} + \frac{1}{\tau}))\lambda_{\mathsf{h}}^{II} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_B(\frac{1}{\mu_{\mathsf{h}} - \lambda_{\mathsf{h}}^{II}} + \frac{1}{\tau}))\lambda_{\mathsf{l}}^{II} \\
&\ge \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_B(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_B(\frac{1}{\mu_{\mathsf{h}} - \lambda_E^*} + \frac{1}{\tau}))\lambda_E^* = \Pi_E^*.
\end{aligned}
$$

The last inequality follows our definitions of $\lambda_t^{II}$ in Part II. □

**Proof of Theorem 5**

We first verify the result for full-timers. As we have mentioned in Section 5.1, the key is to show that the optimal fraction of full-timers to hire $q_H^*$ in the HM equals $q_E^*$ in the EM. To this end,

$$
\begin{aligned}
\frac{\partial \Pi_H}{\partial q} &= \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t(p_t - r_B(\frac{\mu_t - q_{a,t}\lambda_{a,t}}{(\mu_t - q\lambda_f - q_{a,t}\lambda_{a,t})^2} + \frac{1}{\tau}) - c_a \frac{q_{a,t}\lambda_{a,t}}{(\mu_t - q\lambda_f - q_{a,t}\lambda_{a,t})^2})\lambda_f \\
&\le \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t(p_t - r_B(\frac{\mu_t - q_{a,t}\lambda_{a,t}}{(\mu_t - q\lambda_f - q_{a,t}\lambda_{a,t})^2} + \frac{1}{\tau}))\lambda_f = \frac{\partial \Pi_E}{\partial q}.
\end{aligned}
$$

Therefore we have $\partial\Pi_H/\partial q|_{q=q_E^*} \le \partial\Pi_E/\partial q|_{q=q_E^*}$. Together with the concavity of $\Pi_H$ and the constraint $q \ge q_E^*$, this implies that $q_H^* = q_E^*$. Then the transaction volume is clearly higher in the HM than in EM (i.e., $\lambda_H^* \ge \lambda_E^*$) as the company in the HM may also enroll part-timers. Finally, the result for company's profit can be verified as follows,

$$
\begin{aligned}
\Pi_H^* &= \max_{w_{\mathsf{h}}, w_{\mathsf{l}}} \max_{q \ge q_E^*} \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t((p_t - r_B(W_t + \frac{1}{\tau}))q\lambda_f + (p_t - w_t)\lambda_{a,t}) \\
&\ge \max_{q\ge q_E^*} \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t(p_t - r_B(W_t + \frac{1}{\tau}))q\lambda_f = \Pi_E^*. \square
\end{aligned}
$$

**Proof of Proposition 3**

Note that the expected daily transaction volume in the HM equilibrium $\lambda_H^* = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t(q_H^*\lambda_f + q_{a,t}^{H,*}\lambda_{a,t})$. We can verify that

$$\begin{aligned}\frac{\partial^2\Pi_H}{\partial q\partial B} &= \sum_{t\in\{\mathsf{h},\mathsf{l}\}} -\beta_t\left(\frac{\mu_t - q_{a,t}\lambda_{a,t}}{(\mu_t - q\lambda_f - q_{a,t}\lambda_{a,t})^2} + \frac{1}{\tau}\right)\lambda_f \le 0,\\ \frac{\partial^2\Pi_H}{\partial q_{a,t}\partial B} &= -\beta_t\frac{\lambda_{a,t}\cdot q\lambda_f}{(\mu_t - q\lambda_f - q_{a,t}\lambda_{a,t})^2} \le 0,\ t\in\{\mathsf{h},\mathsf{l}\},\end{aligned}$$

which imply that $q_H^*$ and $(q_{a,\mathsf{h}}^{H,*}, q_{a,\mathsf{l}}^{H,*})$ and thus $\lambda_H^*$ will decrease in $B$.

For the company's optimal profit $\Pi_H^*$ in the HM, we have

$$\begin{aligned}\frac{d\Pi_H^*}{dB} &= \frac{\partial\Pi_H^*}{\partial B} + \frac{\partial\Pi_H^*}{\partial q}|_{q=q_E^*}\cdot\frac{\partial q_E^*}{\partial B}\\ &= \sum_{t\in\{\mathsf{h},\mathsf{l}\}} -\beta_t\left(\frac{1}{\mu_t - \lambda_{H,t}^*} + \frac{1}{\tau}\right)q_H^*\lambda_f + \frac{\partial\Pi_H^*}{\partial q}|_{q=q_E^*}\cdot\frac{\partial q_E^*}{\partial B}.\end{aligned} \tag{OA.14}$$

Given that $q_H^* = q_E^* = 1$ when $B = 0$ and that $q_E^*$ will decrease in $B$ as we have just verified (i.e., $d\Pi_H^*/dB \le 0$), there must be some $\overline{B} \ge 0_H$ such that $\partial q_E^*/\partial B = 0$ for $B \le \overline{B}_H$. Then according to (OA.14), we have $d\Pi_H^*/dB = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} -\beta_t(1/(\mu_t - \lambda_{H,t}^*) \le 0$ for $B \le \overline{B}_H$.

Since both $\lambda_H^*$ and $\Pi_H^*$ will decrease in $B$ for $B \le \overline{B}_H$ while both $\lambda^*$ and $\Pi^*$ are constant of $B$, there exist $\overline{B}'_H \le \overline{B}_H$ such that $\lambda_H^* \ge \lambda^*$ and $\Pi_H^* \ge \Pi^*$ for $B \le \overline{B}'_H$. Redefine $\overline{B}_H \equiv \min\{\overline{B}'_H, \overline{B}_H\}$ and we obtain the result. □

**Proof of Corollary 1**

Corollary 1 follows by combining Theorem 1(i) and Proposition 3. □

**Proof of Proposition 4**

The result $S_\pi^* \ge S^*$ naturally holds when $S^* = 0$. When $S^* > 0$, Lemma OA.10 implies that $c_a > r_0$, which leads to the following result.

Lemma. *With $c_a > r_0$, if in the $C^\pi M$ part-timers' participation probability $q_{a,t}^\pi > 0$ in both $t \in \{h, l\}$, the participation probability of committed full-timers (if any) in that period $q_{f,t}^\pi$ must be strictly positive; in particular, we have $q_{f,t}^\pi = 1$.*

Proof. Suppose instead $q_{f,t}^\pi = 0$. Since in period $t$ only part-timers are on the platform (i.e., $q_{a,t}^\pi > 0$) and no full-timer is prioritized over them, the average wait time for a participating part-timer to be dispatched is $1/(\mu_t - q_{a,t}^\pi\lambda_{a,t})$. In addition, the company must have set the earning rate $r_{a,t}^\pi = c_a$ because $r_{a,t}^\pi < c_a$ is too low to incentivize any part-timer while $r_{a,t}^\pi > c_a$ is too much to be optimal even if all part-timers are to be incentivized (i.e., $q_{a,t}^\pi = 1$). Then as Lemma OA.1 implies, the piece-rate wage for period $t$ must be $w_{\pi,t} = c_a(1/(\mu_t - q_{a,t}^\pi\lambda_{a,t}) + 1/\tau)$.

Now, from the perspective of an infinitesimal full-timer, if he or she is to enter the platform, the average wait time to be dispatched is simply $1/\mu_t$ since no other full-timer is present and he or she is prioritized over all part-timers. As such, the average earning rate for him or her will be

$$r^{\pi}_{f,t} = \frac{w_{\pi,t}}{\frac{1}{\mu_t} + \frac{1}{\tau}} = \frac{c_a(\frac{1}{\mu_t - q^{\pi}_{a,t}\lambda_{a,t}} + \frac{1}{\tau})}{\frac{1}{\mu_t} + \frac{1}{\tau}} > c_f,$$

given that $q^{\pi}_{f,t} > 0$ and $c_a > r_0 > c_f$. This implies that so long there are committed full-timers (i.e., $q_{\pi} > 0$), some of them must be willing to participate (i.e., $q^{\pi}_{f,t} > 0$) in period $t$. Indeed, we shall have $q^{\pi}_{f,t} = 1$. Suppose instead $q^{\pi}_{f,t} < 1$. Then the average earning rate for full-timers $r^{\pi}_{f,t}$ must equal $c_f$ as otherwise either more or no committed full-timers will participate. The piece-rate wage for period $t$ is thus $w_{\pi,t} = c_f(1/(\mu_t - q^{\pi}_{f,t} \cdot q_{\pi}\lambda_f) + 1/\tau)$. But then the average earning rate for part-timers becomes

$$r^{\pi}_{a,t} = \frac{w_{\pi,t}}{\frac{1}{(\mu_t - q^{\pi}_{f,t} \cdot q_{\pi}\lambda_f)(\mu_t - q^{\pi}_{f,t} \cdot q_{\pi}\lambda_f - q^{\pi}_{a,t}\lambda_{a,t})/\mu_t} + \frac{1}{\tau}} < c_f < c_a,$$

which renders part-timers' participation probability $q^{\pi}_{a,t} = 0$, which contradicts that $q^{\pi}_{a,t} > 0$. □

Corollary. *With $c_a > r_0$, if in the C$^{\pi}$M part-timers' participation probability $q^{\pi}_{a,t} > 0$ in both $t \in \{\mathsf{h}, \mathsf{l}\}$, full-timers' commitment probability $q_{\pi} = 1$ and their expected utility $u_{\pi} = \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t(r^{\pi}_{f,t} - c_f)^+ > c_a - c_f$.*

Proof. The proof for the previous lemma indicates that if in the C$^{\pi}$M part-timers' participation probability $q^{\pi}_{a,t} > 0$ in some period $t$ and if at the same time also $c_a > r_0$, the average earning rate for full-timers will be

$$r^{\pi}_{f,t} = \frac{w_{\pi,t}}{\frac{1}{\mu_t - q_{\pi}\lambda_f} + \frac{1}{\tau}} = \frac{c_a(\frac{1}{(\mu_t - q_{\pi}\lambda_f)(\mu_t - q_{\pi}\lambda_f - q^{\pi}_{a,t}\lambda_{a,t})/\mu_t} + \frac{1}{\tau})}{\frac{1}{\mu_t - q_{\pi}\lambda_f} + \frac{1}{\tau}} > \frac{c_a(\frac{1}{(\mu_t - q_{\pi}\lambda_f)^2/\mu_t} + \frac{1}{\tau})}{\frac{1}{\mu_t - q_{\pi}\lambda_f} + \frac{1}{\tau}} > c_f. \text{(OA.15)}$$

The expected utility for full-timers in the C$^{\pi}$M is thus

$$u_{\pi} = \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t(r^{\pi}_{f,t} - c_f)^+ > c_a \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \frac{\beta_t(\frac{1}{(\mu_t - q_{\pi}\lambda_f)^2/\mu_t} + \frac{1}{\tau})}{\frac{1}{\mu_t - q_{\pi}\lambda_f} + \frac{1}{\tau}} - c_f > c_a - c_f > u_0. \square$$

The result that $S^*_{\pi} > S^*$ then immediately ensues since $u^* \le c_a - c_f$. To see that $(S^*_{\pi} - S^*)/S^*$ will increase in part-timers' labor pool size $M_a$, the second equality in (OA.15) shows that committed full-timers' earning rates during both peak and off-peak periods (and thus their welfare $S^*_{\pi}$) are increasing in part-timers' arrival rate $\lambda_{a,t}$ (and thus the labor pool size $M_a$), while $S^* = c_a - c_f$, or $\beta_{\mathsf{h}}(c_a - c_f)$ or $\beta_{\mathsf{l}}(c_a - c_f)$ is constant in $M_a$. □

**Proof of Proposition 5**

The proof will proceed in a similar spirit as that for Proposition 1(iii). We will first lay out all possible cases for the company's optimization problem in the CM and then for each case we analyze the sufficient conditions that guarantee both the transaction volume and the company's profit will be higher in the C$^\pi$M than in the CM. Finally, we summarize the discussion and show that all sufficient conditions can be consolidate into that part-timers' opportunity cost $c_a$ is less than some threshold $\underline{c}_\pi$. To streamline the discussion, we restrict the attention to where $c_a < c_f$. According to Lemma OA.4, we consider following cases.

*Case I:* $r_h = r_l = c_a$. In this case, both during peak and off-peak periods part-timers will participate with mixed strategies while no committed full-timers will participate; in particular, no full-timer will commit at all as the expected utility is strictly lower than their outside option (i.e., $u = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (r_t - c_f)^+ < u_0$). The company's optimization problem is thus to control part-timers' participation probabilities $q_{a,\mathsf{h}}$ and $q_{a,\mathsf{l}}$, i.e.,

$$\max_{(q_{a,\mathsf{h}},q_{a,\mathsf{l}})\in[0,1]^2} \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t \Big(p_t - c_a\big(\frac{1}{\mu_\mathsf{h} - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}\big)\Big) q_{a,t}\lambda_{a,t}. \tag{OA.16}$$

In this case, the optimal profit $\Pi^*$ is decreasing in $c_a$, according to the Envelop Theorem.

*Case II:* $r_h = c_a, r_l = (r_0 - \beta_h c_f)/\beta_l$. In this case, during peak part-timers will participate with a mixed strategy while no committed full-timers will participate, and during off-peak periods all part-timers available and committed full-timers will participate. In particular, full-timers will commit with a mixed strategy as the expected utility equals their outside option (i.e., $u = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (r_t - c_f)^+ = u_0$). The company's optimization problem is thus to control part-timers' participation probability $q_{a,\mathsf{h}}$ during peak periods and full-timers' commitment probability $q$, i.e.,

$$\max_{(q_{a,\mathsf{h}},q)\in[0,1]^2} \beta_\mathsf{h}\Big(p_\mathsf{h} - c_a\big(\frac{1}{\mu_\mathsf{h} - q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}} + \frac{1}{\tau}\big)\Big) q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}} + \beta_\mathsf{l}\Big(p_\mathsf{l} - \frac{r_0 - \beta_\mathsf{h} c_f}{\beta_\mathsf{l}}\big(\frac{1}{\mu_\mathsf{l} - q\lambda_f - \lambda_{a,\mathsf{l}}} + \frac{1}{\tau}\big)\Big)(q\lambda_f + \lambda_{a,\mathsf{l}}),$$

which, as one can verify using the Envelope theorem, will decrease in $c_a$.

*Case III:* $r_h = c_f, r_l = (r_0 - \beta_h c_f)/\beta_l$. In this case, during peak all part-timers available will participate while committed full-timers will participate with a mixed strategy, and during off-peak periods all part-timers available and committed full-timers will participate. Again, full-timers will commit with a mixed strategy as the expected utility equals their outside option. The company's optimization problem is thus to control committed full-timers' participation probability $q_{f,\mathsf{h}}$ during peak periods and full-timers' commitment probability $q$, i.e.,

$$\max_{(q_{f,\mathsf{h}},q)\in[0,1]^2} \beta_\mathsf{h}\Big(p_\mathsf{h} - c_f\big(\frac{1}{\mu_\mathsf{h} - \lambda_{a,\mathsf{h}} - q_{f,\mathsf{h}}\cdot q\lambda_f} + \frac{1}{\tau}\big)\Big)(\lambda_{a,\mathsf{h}} + q_{f,\mathsf{h}}\cdot q\lambda_f) + \beta_\mathsf{l}\Big(p_\mathsf{l} - \frac{r_0 - \beta_\mathsf{h} c_f}{\beta_\mathsf{l}}\big(\frac{1}{\mu_\mathsf{l} - \lambda_{a,\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}\big)\Big)(\lambda_{a,\mathsf{l}} + q\lambda_f).$$

*Case IV:* $r_h \in (c_f, (r_0 - \beta_l c_f)/\beta_h), r_l = (r_0 - \beta_h r_h)/\beta_l$. In this case, both during peak and off-peak periods all part-timers available and committed full-timers will participate, and full-timers will commit with a mixed strategy as the expected utility again equals their outside option. The company's optimization problem is thus to control full-timers' commitment probability $q$, i.e.,

$$\max_{r_{\mathsf{h}} \in (c_f, \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}), q \in [0,1]} \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - \lambda_{a,\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))(\lambda_{a,\mathsf{h}} + q\lambda_f) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}}}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - \lambda_{a,\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}))(\lambda_{a,\mathsf{l}} + q\lambda_f).$$

*Case V:* $r_h = (r_0 - \beta_l c_f)/\beta_h, r_l = c_f$. In this case, during peak periods all part-timers available and committed full-timers will participate, and during off-peak periods all part-timers available will participate while committed full-timers will participate with a mixed strategy. Again, full-timers will commit with a mixed strategy as the expected utility equals their outside option. The company's optimization problem is thus to control committed full-timers' participation probability $q_{f,\mathsf{l}}$ during off-peak periods and full-timers' commitment probability $q$, i.e.,

$$\max_{(q_{f,\mathsf{l}}, q) \in [0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}} - \lambda_{a,\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))(\lambda_{a,\mathsf{h}} + q\lambda_f) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_f(\frac{1}{\mu_{\mathsf{l}} - \lambda_{a,\mathsf{l}} - q_{f,\mathsf{l}} \cdot q\lambda_f} + \frac{1}{\tau}))(\lambda_{a,\mathsf{l}} + q_{f,\mathsf{l}} \cdot q\lambda_f).$$

*Case VI:* $r_h = (r_0 - \beta_l c_f)/\beta_h, r_l = c_a$. In this case, during peak periods all part-timers available and committed full-timers will participate, and during off-peak periods part-timers will participate with a mixed strategy while no committed full-timers will participate. Again, full-timers will commit with a mixed strategy as the expected utility equals their outside option. The company's optimization problem is thus to control part-timers' participation probability $q_{a,\mathsf{l}}$ during off-peak periods and full-timers' commitment probability $q$, i.e.,

$$\max_{(q_{a,\mathsf{l}}, q) \in [0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}} - \lambda_{a,\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))(\lambda_{a,\mathsf{h}} + q\lambda_f) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - q_{a,\mathsf{l}} \cdot \lambda_{a,\mathsf{l}}} + \frac{1}{\tau}))q_{a,\mathsf{l}} \cdot \lambda_{a,\mathsf{l}},$$

which, as one can again verify using the Envelope theorem, will decrease in $c_a$.

Now, following a similar argument as in the Proof of Proposition 1(iii), one can verify that the company's profit $\Pi^*$ in the CM equilibrium will either decrease or constant in $c_a$ for all cases. In particular, for Cases I, II and VI where $\Pi^*$ is decreasing in $c_a$, one verify that Case I renders the decline in $\Pi^*$ the steepest (i.e., $d\Pi^*/dc_a$ is the lowest). As such, there shall exist $\underline{c}_\pi$ such that for $c_a \leq \underline{c}_\pi$, we will have Case I prevail in the CM equilibrium. As we will show below, that Case I prevails will facilitate the analysis for the results on the transaction volume.

Next, we discuss the sufficient conditions to ensure that both the equilibrium transaction volume and the company's optimal profit will be higher in the C$^\pi$M than in the CM. The key to the analysis is the company's operational strategies in the C$^\pi$M. Because now full-timers are prioritized over part-timers, the average wait times to be dispatched for these two groups of workers differ, so in any period $t$, the same piece-rate wage $w_{\pi,t}$ will yield different average earning rates. Similarly as in the CM, with a little abuse of notation we denote the *submarket equilibrium* among full- and part-timers in the C$^\pi$M as $(\mathcal{Q}_{\pi,f}, \mathcal{R}_{\pi,f})$ and $(\mathcal{Q}_{\pi,a}, \mathcal{R}_{\pi,a})$, respectively. Then similarly as in Lemma OA.2, we can verify that the company's problem in the C$^\pi$M is essentially to either control $(\mathcal{Q}_{\pi,f}, \mathcal{R}_{\pi,f})$ or $(\mathcal{Q}_{\pi,a}, \mathcal{R}_{\pi,a})$. As such, one can analyze the company's optimization problem in the C$^\pi$M similarly as in the CM by considering all possible cases (i.e., values of any submarket equilibrium) that may prevail in the C$^\pi$M equilibrium. In particular, one can verify that given same set of model primitives, what will prevail in the C$^\pi$M essentially coincide with those in the CM; that is, the possible cases of market equilibrium in the CM will exhaust all possible values a submarket equilibrium in the C$^\pi$M can potentially take, and vice versa. Therefore, given that with $c_a \le \underline{c}_\pi$ Case I will prevail in the CM equilibrium, to study the sufficient conditions for the company's optimal profit in the C$^\pi$M to be higher than in the CM (i.e., $\Pi_\pi^* \ge \Pi^*$), it suffices to investigate under what circumstances $\Pi_\pi^* \ge \Pi^*$ should Case I also prevail for the C$^\pi$M.

Now, denote the optimal solution to (OA.16) by $q_{a,\mathsf{h}}^*$ and $q_{a,\mathsf{l}}^*$. The corresponding piece-rate wages are thus $w_t^* = c_a(1/(\mu_t - q_{a,t}^*\lambda_{a,t}) + 1/\tau)$ for $t \in \{\mathsf{h}, \mathsf{l}\}$. Suppose $w_t^* \le c_f(1/\mu_t + 1/\tau)$, or equivalently, $c_a \le \underline{c}'_\pi \equiv c_f(1/\mu_t + 1/\tau)/(1/(\mu_t - q_{a,t}^*\lambda_{a,t}) + 1/\tau)$. Because full-timers are prioritized in the C$^\pi$M, the average wait time to be dispatched in period $t$ is $1/(\mu_t - q_t \cdot q\lambda_f)$ should $q_t \cdot q\lambda_f$ many full-timers participate. Therefore, the piece-rate wage shall be strictly higher than $c_f(1/\mu_t + 1/\tau)$ to incentivize a positive number of full-timers. Yet with the condition $c_a \le \underline{c}'_\pi$, if the company sets piece-rate wages in the C$^\pi$M $w_{\pi,t} = w_t^*$ for $t \in \{\mathsf{h}, \mathsf{l}\}$, then clearly no full-timer will ever commit or participate; on the other hand though, part-timers will participate up to the same level as they do in the CM (i.e., $q_{a,t}^\pi = q_{a,t}^*$) since there is no full-timer to be prioritized over them at all. Hence, with $w_{\pi,t} = w_t^*$ for $t \in \{\mathsf{h}, \mathsf{l}\}$, the company will make the same amount of profit in the C$^\pi$M as in the CM, and therefore we must have $\Pi_\pi^* \ge \Pi^*$.

Finally, we study the conditions for the transaction volume to be higher in the C$^\pi$M than in the CM (i.e., $\lambda_\pi^* \ge \lambda^*$). If Case I also prevails in the C$^\pi$M, then clearly we have $\lambda_\pi^* \ge \lambda^*$. Suppose instead Case II is to prevail in the C$^\pi$M. If $c_a \le c_f(1/\mu_\mathsf{h} + 1/\tau)/(1/(\mu_\mathsf{h} - q_{a,\mathsf{h}}^*\lambda_{a,\mathsf{h}}) + 1/\tau)$, then as we have analyzed previously during peak periods we shall have the transaction volume higher in the C$^\pi$M than in the CM (i.e., $\lambda_{\pi,t}^* \ge \lambda_t^*$). Then we denote the optimal commitment probability among full-timers in the CM by $q^*$. If

$$c_a \le \underline{c}''_\pi \equiv \frac{\frac{r_0 - \beta_\mathsf{h} c_f}{\beta_\mathsf{l}}\left(\frac{1}{\mu_\mathsf{l}} + \frac{1}{\tau}\right)}{\frac{1}{\mu_\mathsf{l} - \lambda_{a,\mathsf{l}}} + \frac{1}{\tau}},$$

then by setting the piece-rate wage during off-peak periods as $w_{\pi,\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}(1/(\mu_{\mathsf{l}} - q\lambda_f) + 1/\tau)$ for any $q \in [0,1]$ in the C$^\pi$M, the transaction volume will be $\lambda_{\pi,\mathsf{l}} = q\lambda_f + \lambda_{a,\mathsf{l}}$, which is higher than that $\lambda_{\mathsf{l}}^*$ in the CM. Combining the results during both peak and off-peak periods, we have $\lambda_\pi^* \geq \lambda^*$. One can similarly verify that for the remaining case we also have $\lambda_\pi^* \geq \lambda^*$ for sufficiently small $c_a$.

In summary, we have shown above that when $c_a \leq \min\{\underline{c}_\pi, \underline{c}'_\pi\}$, we have the company's profit higher in the C$^\pi$M than in the CM; in addition, when $c_a \leq \underline{c}''_\pi$, we also have $\lambda_\pi^* \geq \lambda^*$. Putting together, redefine $\underline{c}_\pi$ as the minimum among the original $\underline{c}_\pi$ and $\underline{c}'_\pi$ and $\underline{c}''_\pi$, we obtain the results in Proposition 5. $\square$

**Proof of Proposition 6**

**Proof of Proposition 6(i).** Define $\bar{\bar{c}} = (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$, which is strictly higher (a) than $(r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$ given that $\beta_{\mathsf{l}} > \beta_{\mathsf{h}}$ and (b) than $c_f$ since $r_0 > c_f$. When $c_a \geq (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$, part-timers' welfare $\tilde{S}^*$ in the CM equilibrium must equal 0. To see this, first note that par-timers' welfare is defined as $\tilde{S} = M_a \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t \gamma_{a,t}(r_t - c_a)^+$. If $\tilde{S}^* > 0$, the company must have set the optimal average earning rate $r_{t'}^* > c_a$ for some $t' \in \{\mathsf{h},\mathsf{l}\}$. As such, part-timers available during period $t'$ will all participate and in addition, all full-timers will commit and also participate during period $t'$. The latter is because (a) full-timers' expected utility $u = \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t (r_t^* - c_f)^+ \geq \beta_{t'}(r_{t'}^* - c_f)^+ > u_0$ and (b) that $r_{t'}^* > c_f$. Then clearly the company can earn a strictly higher profit by selecting a strictly lower average earning rate $r_{t'}^* - \epsilon$ such that still the same amount of workers, part- or full-time, will participate. Contradiction.

Next, we show that when $c_a < (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$, part-timers' welfare $\tilde{S}^* > 0$ in the CM equilibrium if their labor pool size $M_a$ is lower than some threshold $\bar{\bar{M}}$. We elaborate the idea with the case where $c_a \in (r_0, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$. The remaining scenarios (i.e., $c_a < r_0$) can be verified similarly.

Recall our discussion in the Proof of Proposition 1(i). We have the following conclusion.

Lemma *For $c_a \in (r_0, (r_0 - \beta_h c_f)/\beta_l)$, if $\tilde{S}^* = 0$, the company's optimal profit $\Pi^*$ must be*

$$\tilde{\Pi}^{*,0} \equiv \max\{ \quad \Pi_1^* \equiv \max_{(q_{a,h},q_{a,l}) \in [0,1]^2} \sum_{t \in \{h,l\}} \beta_t (p_t - c_a(\tfrac{1}{\mu_t - \lambda_f - q_{a,t}\lambda_{a,t}} + \tfrac{1}{\tau}))(\lambda_f + q_{a,t}\lambda_{a,t}),$$

$$\Pi_2^* \equiv \max_{(q,q_{a,l}) \in [0,1]^2} \beta_h (p_h - \tfrac{r_0 - \beta_l c_a}{\beta_h}(\tfrac{1}{\mu_h - q\lambda_f} + \tfrac{1}{\tau}))q\lambda_f + \beta_l(p_l - c_a(\tfrac{1}{\mu_l - q\lambda_f - q_{a,l}\lambda_{a,l}} + \tfrac{1}{\tau}))(q\lambda_f + q_{a,l}\lambda_{a,l}),$$

$$\Pi_3^* \equiv \max_{(q,q_{a,h}) \in [0,1]^2} \beta_h (p_h - c_a(\tfrac{1}{\mu_h - q\lambda_f - q_{a,h}\lambda_{a,h}} + \tfrac{1}{\tau}))(q\lambda_f + q_{a,h}\lambda_{a,h}) + \beta_l(p_l - \tfrac{r_0 - \beta_h c_a}{\beta_l}(\tfrac{1}{\mu_l - q\lambda_f} + \tfrac{1}{\tau}))q\lambda_f\}.$$

*If instead $\tilde{S}^* > 0$ in the CM, the company's optimal profit $\Pi^*$ must be*

$$\tilde{\Pi}^{*,+} \equiv \max\{\Pi_4^* \equiv \max_{r_h \in (c_a, \frac{r_0 - \beta_l c_f}{\beta_h}), q \in [0,1]} \beta_h(p_h - r_h(\tfrac{1}{\mu_h - q\lambda_f - \lambda_{a,h}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,h}) + \beta_l(p_l - \tfrac{r_0 - \beta_h r_h}{\beta_l}(\tfrac{1}{\mu_l - q\lambda_f} + \tfrac{1}{\tau}))q\lambda_f,$$

$$\max_{(q,q_{f,l})\in[0,1]^2} \beta_h(p_h - \frac{r_0-\beta_l c_f}{\beta_h}(\frac{1}{\mu_h - q\lambda_f - \lambda_{a,h}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,h}) + \beta_l(p_l - c_f(\frac{1}{\mu_l - q_{f,l}\cdot q\lambda_f} + \frac{1}{\tau}))q_{f,l}\cdot q\lambda_f\}.$$

Proof. See Lemma OA.3 and the Proof. One may observe that the optimal earning rate in off-peak periods $r_\mathsf{l}^*$ is always less than or equal to $c_a$. As such, part-timers' welfare (i.e., $\tilde{S}^* > 0$) is positive in the CM equilibrium if and only if the optimal earning rate in peak periods $r_\mathsf{h}^* > c_a$. □

To derive the sufficient condition for $\tilde{\Pi}^{*,+} \geq \tilde{\Pi}^{*,0}$ (and thus $\tilde{S}^* > 0$), it is sufficient to study that for $\Pi_4^* \geq \tilde{\Pi}^{*,0}$, which in turn is equivalent to $\Pi_4^* \geq \Pi_k^*$ for $k \in \{1,2,3\}$. First, for $\Pi_4^*$ to be higher than $\Pi_3^*$, denote by $r_\mathsf{l} = (r_0 - \beta_\mathsf{h} c_a)/\beta_\mathsf{l}$. We show in the Proof of Proposition 1(iii) that when $\lambda_{a,\mathsf{h}} < \bar{\lambda}_a \equiv (\mu_\mathsf{h} - \sqrt{\mu_\mathsf{h}\tau c_a/(p_\mathsf{h}\tau - c_a)}) - (\mu_\mathsf{l} - \sqrt{\mu_\mathsf{l}\tau r_\mathsf{l}/(p_\mathsf{h}\tau - r_\mathsf{l})})$ where $r_\mathsf{l} = (r_0 - \beta_\mathsf{h} c_a)/\beta_\mathsf{l}$, we have $q_{a,\mathsf{h}}^* = 1$. Denote by $q_3^*$ the optimal commitment probability among full-timers for $\Pi_3^*$ given that $\lambda_{a,\mathsf{h}} < \bar{\lambda}_a$. Further denote by $r_4^*$ the optimal average earning rate $r_\mathsf{h}^*$ during peak periods for $\Pi_4^*$. Then when $\lambda_{a,\mathsf{h}} < \bar{\lambda}_a$ we have

$$\begin{aligned}\Pi_4^* - \Pi_3^* &\geq \beta_\mathsf{h}(c_a - r_4^*)(\frac{1}{\mu_\mathsf{h} - q_3^*\lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau})(q_3^*\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_\mathsf{l}(\frac{r_0 - \beta_\mathsf{h} c_a}{\beta_\mathsf{l}} - \frac{r_0 - \beta_\mathsf{h} r_4^*}{\beta_\mathsf{l}})(\frac{1}{\mu_\mathsf{l} - q_3^*\lambda_f} + \frac{1}{\tau})q_3^*\lambda_f \\ &\geq \beta_\mathsf{h}(r_4^* - c_a)((\frac{1}{\mu_\mathsf{l} - q_3^*\lambda_f} + \frac{1}{\tau})q_3^*\lambda_f - (\frac{1}{\mu_\mathsf{h} - q_3^*\lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau})(q_3^*\lambda_f + \lambda_{a,\mathsf{h}})). \qquad \text{(OA.17)}\end{aligned}$$

Note that (OA.17) is positive at $\lambda_{a,\mathsf{h}} = 0$ and is decreasing in $\lambda_{a,\mathsf{h}}$. Hence there exists a $\bar{\lambda}_a'$ such that if $\lambda_{a,\mathsf{h}} \leq \bar{\lambda}_a'$ we have $\Pi_4^* \geq \Pi_3^*$.

Next, for $\Pi_4^*$ to be higher than $\Pi_2^*$. Following a similar argument as in the Proof of Proposition 1(iii), one can verify that the optimal participation probability among part-timers during off-peak periods for $\Pi_2^*$ is $q_{a,\mathsf{l}}^* = 0$. Then the optimal commitment probability among full-timers for $\Pi_2^*$, which we denote by $q_2^*$, is given by the first-order condition

$$\beta_\mathsf{h}(p_\mathsf{h} - \frac{r_0 - \beta_\mathsf{l} c_a}{\beta_\mathsf{h}}(\frac{\mu_\mathsf{h}}{(\mu_\mathsf{h} - q_2^*\lambda_f)^2} + \frac{1}{\tau})) + \beta_\mathsf{l}(p_\mathsf{l} - c_a(\frac{\mu_\mathsf{l}}{(\mu_\mathsf{l} - q_2^*\lambda_f)^2} + \frac{1}{\tau})) = 0. \qquad \text{(OA.18)}$$

Because $p_\mathsf{h} > p_\mathsf{l}$ and $c_a > r_0$, one can verify that the first summand in (OA.18) is strictly higher than the second summand, which implies that

$$\beta_\mathsf{h}(p_\mathsf{h} - \frac{r_0 - \beta_\mathsf{l} c_a}{\beta_\mathsf{h}}(\frac{\mu_\mathsf{h}}{(\mu_\mathsf{h} - q_2^*\lambda_f)^2} + \frac{1}{\tau})) > 0 > \beta_\mathsf{l}(p_\mathsf{l} - c_a(\frac{\mu_\mathsf{l}}{(\mu_\mathsf{l} - q_2^*\lambda_f)^2} + \frac{1}{\tau})).$$

The first inequality indicates the existence of a $\bar{\lambda}_a'$ that for $\lambda_{a,\mathsf{h}} \leq \bar{\lambda}_a'$ we have

$$\begin{aligned}\Pi_2^* &= \beta_\mathsf{h}(p_\mathsf{h} - \frac{r_0 - \beta_\mathsf{l} c_a}{\beta_\mathsf{h}}(\frac{1}{\mu_\mathsf{h} - q_2^*\lambda_f} + \frac{1}{\tau}))q_2^*\lambda_f + \beta_\mathsf{l}(p_\mathsf{l} - c_a(\frac{1}{\mu_\mathsf{l} - q_2^*\lambda_f} + \frac{1}{\tau}))q_2^*\lambda_f \\ &\leq \beta_\mathsf{h}(p_\mathsf{h} - \frac{r_0 - \beta_\mathsf{l} c_a}{\beta_\mathsf{h}}(\frac{1}{\mu_\mathsf{h} - (q_2^*\lambda_f + \lambda_{a,\mathsf{h}})} + \frac{1}{\tau}))(q_2^*\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_\mathsf{l}(p_\mathsf{l} - c_a(\frac{1}{\mu_\mathsf{l} - q_2^*\lambda_f} + \frac{1}{\tau}))q_2^*\lambda_f.\end{aligned}$$

As such, given $\lambda_{a,\mathsf{h}} \leq \bar{\lambda}_a'$,

$$\Pi_4^* - \Pi_2^* \geq \beta_\mathsf{h}(\frac{r_0 - \beta_\mathsf{l} c_a}{\beta_\mathsf{h}} - r_4^*)(\frac{1}{\mu_\mathsf{h} - q_2^*\lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau})(q_2^*\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_\mathsf{l}(c_a - \frac{r_0 - \beta_\mathsf{h} r_4^*}{\beta_\mathsf{l}})(\frac{1}{\mu_\mathsf{l} - q_2^*\lambda_f} + \frac{1}{\tau})q_2^*\lambda_f$$

$$\geq (\beta_{\mathsf{h}} r_4^* + \beta_{\mathsf{l}} c_a - r_0)((\frac{1}{\mu_{\mathsf{l}} - q_2^* \lambda_f} + \frac{1}{\tau}) q_2^* \lambda_f - (\frac{1}{\mu_{\mathsf{h}} - q_2^* \lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau})(q_2^* \lambda_f + \lambda_{a,\mathsf{h}})). \tag{OA.19}$$

Similarly we have shown when comparing $\Pi_4^*$ and $\Pi_3^*$, here (OA.19) is also decreasing in $\lambda_{a,\mathsf{h}}$ and strictly positive at $\lambda_{a,\mathsf{h}} = 0$. If (OA.19) is positive at $\lambda_{a,\mathsf{h}} = \bar{\lambda}'_a$, then we must have $\Pi_4^* \geq \Pi_2^*$ for all $\lambda_{a,\mathsf{h}} \leq \bar{\lambda}'_a$; otherwise, we redefine $\bar{\lambda}'_a$ as the root of (OA.19) and will then again have $\Pi_4^* \geq \Pi_2^*$ for $\lambda_{a,\mathsf{h}} \leq \bar{\lambda}'_a$.

Finally, we compare $\Pi_4^*$ and $\Pi_1^*$. One can verify that the optimal participation probability among part-timers during period $t$ for $\Pi_1^*$ is

$$q_{a,t}^* = \min\{\frac{(\mu_t - \sqrt{\frac{\mu_t \tau c_a}{p_t \tau - c_a}} - \lambda_f)^+}{\lambda_{a,t}}, 1\},\ t \in \{\mathsf{h}, \mathsf{l}\}.$$

Hence, for $\lambda_{a,\mathsf{h}} \leq (\mu_t - \sqrt{\mu_t \tau c_a / (p_t \tau - c_a)} - \lambda_f)^+$, we have $q_{a,\mathsf{h}}^* = 1$ and the corresponding first-order condition is

$$p_{\mathsf{h}} - c_a(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \lambda_f - \lambda_{a,\mathsf{h}})^2} + \frac{1}{\tau}) \geq 0.$$

As such, by applying the Envelope Theorem, we have

$$\frac{\partial \Pi_1^*}{\partial \lambda_{a,\mathsf{h}}} = \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_a(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \lambda_f - \lambda_{a,\mathsf{h}})^2} + \frac{1}{\tau})) \geq 0,$$

that is, $\Pi_1^*$ is increasing in part-timers' arrival rate during peak periods. One can similarly verify that $\partial \Pi_1^* / \partial \lambda_{a,\mathsf{l}} \geq 0$.

Now, the second summand in $\Pi_4^*$ is independent of part-timers' arrival rates. For the first summand in $\Pi_4^*$, the proof of Proposition 1(iii) implies that when $\lambda_{a,\mathsf{h}} < \bar{\lambda}_a$, which we defined previously when comparing $\Pi_4^*$ and $\Pi_3^*$, we shall have

$$\frac{\partial \Pi_4^*}{\partial \lambda_{a,\mathsf{h}}} = \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_4^*(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - q_4^* \lambda_f - \lambda_{a,\mathsf{h}})^2} + \frac{1}{\tau})) \geq 0,$$

where $q_4^*$ denotes the optimal commitment probability among full-timers for $\Pi_4^*$. Notice that $r_4^* > c_a$ and that $q_4^* \leq 1$ is decreasing in $\lambda_{a,\mathsf{h}}$. As such, for sufficiently low $\lambda_{a,\mathsf{h}}$ we shall have $\partial \Pi_1^* / \partial \lambda_{a,\mathsf{h}} \geq \partial \Pi_4^* / \partial \lambda_{a,\mathsf{h}} \geq 0$. Then putting all together, there exists a $\bar{\lambda}''_a$ such that when $\lambda_{a,\mathsf{h}} \leq \bar{\lambda}''_a$ and $\lambda_{a,\mathsf{l}} \leq \bar{\lambda}''_a$, we have $\Pi_4^* \geq \Pi_1^*$.

In summary, we have shown that for $\lambda_{a,\mathsf{h}} \leq \bar{\lambda}'''_a \equiv \min\{\bar{\lambda}_a, \bar{\lambda}'_a, \bar{\lambda}''_a\}$ and $\lambda_{a,\mathsf{l}} \leq \bar{\lambda}''_a$, we have $\Pi_4^* \geq \tilde{\Pi}^{*,0}$ and thus the part-timers' welfare $\tilde{S}^* \geq 0$. Since $\lambda_{a,t} = \gamma_{a,t} M_a \tau$ for $t \in \{\mathsf{h}, \mathsf{l}\}$, the existence of the threshold $\tilde{\bar{M}}$ follows immediately. □

**Proof of Proposition 6(ii).** That part-timers' welfare equals 0 in the EM equilibrium (i.e., $\tilde{S}_E^* = 0$) is straightforward as we have assumed that they will exit the market in the EM (due to the company's strict workforce control). That their welfare also equals 0 in the HM equilibrium (i.e., $\tilde{S}_H^* = 0$) is also rather intuitive. Note that, part-timers' welfare is defined as $\tilde{S}_H = M_a \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t \gamma_{a,t}(r_{H,t} - c_a)^+$, where $r_{H,t}$ denotes the average earning rate in period $t$. That $\tilde{S}_H^* > 0$ implies $r_{H,t} > c_a$ for some $t$, which in turn indicates that all part-timers' will choose to participate. But then the company can obviously make a higher profit by strictly lowering $r_{H,t}$ such that still all part-timers' will participate; in particular, a lower $r_{H,t}$ will not impair full-timers' incentives to participation as they are treated as employees and guaranteed a flat earning rate (i.e., $r_B$). $\square$

**Proof of Proposition 6(iii).** See Proof of Theorem 2 Part I for the existences of $\underline{B}_+$ and $\overline{B}_+$. Following a similar argument as in the Part I of the Proof of Theorem 2, one can verify that part-timers' welfare is higher in the C$^+$M than in the CM (i.e., $\tilde{S}_+^* \geq \tilde{S}^*$). To show that $(\tilde{S}_+^* - \tilde{S}^*)/\tilde{S}^*$ will decrease in part-timers' labor pool size $M_a$ given that $\tilde{S}^* > 0$, below we again use the case with $c_a \in (r_0, (r_0 - \beta_\mathsf{h} c_f)/\beta_\mathsf{l})$ to illustrate the idea. The remaining scenarios can be verified similarly.

The proof of Proposition 1(i) hints that when $c_a \in (r_0, (r_0 - \beta_\mathsf{h} c_f)/\beta_\mathsf{l})$, if part-timers' surplus $\tilde{S}^* > 0$ in the CM equilibrium, the company must have set the optimal earning rates as $r_\mathsf{h}^* = r_\mathsf{h}$ for some $r_\mathsf{h} \in (c_a, (r_0 - \beta_\mathsf{l} c_f)/\beta_\mathsf{h}]$ and $r_\mathsf{l}^* = (r_0 - \beta_\mathsf{h} r_\mathsf{h}^*)/\beta_\mathsf{l}$ (which is strictly less than $c_a$). Recall that $w_\mathsf{h}^*$ and $w_\mathsf{l}^*$ denote the corresponding optimal piece-rate wages in the CM. Suppose first that $B$ is sufficiently low such that $r_B/\tau \leq \min_{t\in\{\mathsf{h},\mathsf{l}\}} w_t^*$. Then clearly the optimal piece-rate wages in the C$^+$M are the same as in the CM, and so are the corresponding average earning rates, i.e., $r_{+,\mathsf{h}}^* = r_\mathsf{h}^*$ and $r_\mathsf{l}^* = r_{+,\mathsf{l}}^*$. Then suppose $r_B/\tau > w_\mathsf{h}^*$ while $r_B/\tau \leq w_\mathsf{l}^*$, which imply that the off-peak wage $w_\mathsf{l}^*$ in the CM is still feasible (and thus still optimal) in the C$^+$M while the peak wage $w_\mathsf{h}^*$ is no longer so. To pin down the optimal piece-rate wage during peak periods for the C$^+$M, notice that in this case part-timers who are available during peak periods will all participate, and the company is controlling full-timers' commitment probability $q$; in particular, the company's profit function (see the Lemma in the Proof of Proposition 1(i)) is concave in full-timers' commitment probability $q$. This implies that the constraint $w_\mathsf{h} \geq r_B/\tau$ will push up $q$ until it reaches 1; that is, the optimal peak wage $w_{+,\mathsf{h}}^*$ will equal $r_B/\tau$, the corresponding average earning rate $r_{+,\mathsf{h}}^*$ will still be $r_\mathsf{h}^*$, and the optimal commitment probability $q_+^*$ be such that $w_{+,\mathsf{h}}^* = r_B/\tau$. If $q$ has already reach 1, then all full-timers have been attracted to the platform. Hence the corresponding average earning rate $r_{+,\mathsf{h}}^* = (r_B/\tau)/(1/(\mu_\mathsf{h} - \lambda_f - \lambda_{a,t}) + 1/\tau)$. In summary, we have

$$r_{+,\mathsf{h}}^* = \begin{cases} r_\mathsf{h}^* & \text{if } r_B \leq \tau \cdot r_\mathsf{h}^*\left(\frac{1}{\mu_\mathsf{h} - \lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}\right), \\ \frac{r_B/\tau}{\frac{1}{\mu_\mathsf{h} - \lambda_f - \lambda_{a,t}} + \frac{1}{\tau}} & \text{otherwise,} \end{cases}$$

which can be verified to decrease in part-timers' gross arrival rate $\lambda_{a,t}$ and thus their labor pool size $M_a$.

For the remaining cases (i.e., the one with $r_B/\tau \le w^*_{\mathsf{h}}$ but $r_B/\tau > w^*_{\mathsf{l}}$ and the one with $r_B/\tau > w^*_t$ for $t \in \{\mathsf{h},\mathsf{l}\}$), one can similarly verify that the optimal average earning rates $r^*_{+,\mathsf{h}}$ and $r^*_{+,\mathsf{l}}$ in the C$^+$M will decrease in $M_a$. Therefore, part-timers' expected utility $\tilde{u}^*_+$ in the C$^+$M equilibrium must be decreasing in $M_a$ as well. We thus have $(\tilde{S}^*_+ - \tilde{S}^*)/\tilde{S}^* = \tilde{u}^*_+/\tilde{u}^* - 1$ decreasing in $M_a$, given that $\tilde{u}^*$ is constant of $M_a$. □

**Proof of Proposition 6(iv)** The proof hinges on the following result.

LEMMA. *For part-timers' welfare $\tilde{S}^*_\pi > 0$ in the C$^\pi$M equilibrium, there must be at least one period $t \in \{h, l\}$ in which (a) full-timers' participation probability $q^{\pi,*}_{f,t} \cdot q^*_\pi > 0$ and (b) the equilibrium piece-rate wage for that period $w^*_{\pi,t}$ satisfies*

$$w^*_{\pi,t} = r^{\pi,*}_{f,t}\Big(\frac{1}{\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f} + \frac{1}{\tau}\Big) > c_a\Big(\frac{1}{(\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f)(\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f - \lambda_{a,t})/\mu_t} + \frac{1}{\tau}\Big),$$

*where full-timers' average earning rate $r^{\pi,*}_{f,t} \ge c_f$.*

PROOF. Suppose instead there is no such a period in which both conditions are met. Specifically, suppose first in any period $t \in \{\mathsf{h},\mathsf{l}\}$ only condition (b) is satisfied. In other words, in the C$^\pi$M equilibrium only part-timers are enrolled. Then clearly it is not optimal for the company to leave part-timers with $\tilde{S}^*_\pi > 0$. The company can lower the wage $w^*_{\pi,t}$ in some $t \in \{\mathsf{h},\mathsf{l}\}$ so that it is just incentive compatible for part-timers to participate, i.e., $\tilde{S}^*_\pi = 0$. Contradiction. □

Then suppose that in any period $t \in \{\mathsf{h},\mathsf{l}\}$ only condition (a) is satisfied. Then part-timers' average earning rate in any period will be

$$r^{\pi,*}_{a,t} = \frac{w^*_{\pi,t}}{\frac{1}{(\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f)(\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f - \lambda_{a,t})/\mu_t} + \frac{1}{\tau}} \le c_a,$$

which implies that their welfare $\tilde{S}^*_\pi = M_a \sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t \gamma_{a,t} (r^{\pi,*}_{a,t} - c_a)^+ = 0$.

Finally, suppose for any period $t \in \{\mathsf{h},\mathsf{l}\}$, either only one of the conditions is satisfied or neither condition will hold. Following the arguments above we can find that again this will lead to $\tilde{S}^*_\pi = 0$. Contradiction. □

COROLLARY. *For part-timers' welfare $\tilde{S}^*_\pi > 0$ in the C$^\pi$M equilibrium, there must be at least one period $t \in \{h, l\}$ in which their average earning rate*

$$r^{\pi,*}_{a,t} = \frac{r^{\pi,*}_{f,t}\big(\frac{1}{\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f} + \frac{1}{\tau}\big)}{\frac{1}{(\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f)(\mu_t - q^{\pi,*}_{f,t} \cdot q^*_\pi \lambda_f - \lambda_{a,t})/\mu_t} + \frac{1}{\tau}} > c_a,$$

*where full-timers' average earning rate $r^{\pi,*}_{f,t} \ge c_f$.*

Notice that the ratio

$$\frac{\big(\frac{1}{\mu_t - x} + \frac{1}{\tau}\big)}{\frac{1}{(\mu_t - x)(\mu_t - x - \lambda_{a,t})/\mu_t} + \frac{1}{\tau}}$$

is decreasing in $x$. Also, our Proof of Proposition 1 implies that $r_{f,t}^{\pi,*}$ is at most $(r_0 - \beta_{-t}c_f)/\beta_t$, since any higher earning rate is more than enough for it to be incentive compatible for full-timers to commit and participate. As such, part-timers' average earning rate in any period $t$ must be upper bounded as follows,

$$r_{a,t}^{\pi,*} \leq \frac{\frac{r_0-\beta_{-t}c_f}{\beta_t} \cdot (\frac{1}{\mu_t} + \frac{1}{\tau})}{\frac{1}{\mu_t - \lambda_{a,t}} + \frac{1}{\tau}}.$$

Now, define

$$\bar{\bar{c}}_\pi \equiv \max_{t \in \{\mathsf{h},\mathsf{l}\}} \frac{r_0 - \beta_{-t}c_f}{\beta_t} \frac{1/\mu_t + 1/\tau}{1/(\mu_t - \lambda_{a,t}) + 1/\tau}.$$

If $c_a \geq \bar{\bar{c}}_\pi$, then there is no such a period $t \in \{\mathsf{h},\mathsf{l}\}$ in which the condition specified in the Corollary above can be met. Then by contradiction, we must have $\tilde{S}_\pi^* = 0$. By definition, $\bar{\bar{c}}_\pi$ is lower than $\bar{\bar{c}}_\pi$ we have defined in the proof of Proposition 6(i). Since part-timers' welfare in the CM equilibrium $\tilde{S}^*$ can still be positive for $c_a \in [\bar{\bar{c}}, \bar{\bar{c}}_\pi]$ (see details in the Proof of Proposition 6(i)), we have $\tilde{S}^* \geq \tilde{S}_\pi^*$.
□

**Proof of Proposition 7**

**Proof of Proposition 7(i)** The result immediately follows Proposition 6(ii), Theorem 5 and Proposition 2.

**Proof of Proposition 7(ii)** See Proof of Theorem 2 Part I for the existences of $\underline{B}_+$ and $\overline{B}_+$. For $\mathtt{S}^* > 0$, by definition we have

$$\frac{\mathtt{S}_+^* - \mathtt{S}^*}{\mathtt{S}^*} = \frac{M_f u_+^* + M_a \tilde{u}_+^*}{M_f u^* + M_a \tilde{u}^*} - 1 = \frac{\gamma u_+^* + (1-\gamma)\tilde{u}_+^*}{\gamma u^* + (1-\gamma)\tilde{u}^*} - 1. \quad \text{(OA.20)}$$

Following the proofs of Theorem 2(iii) and Proposition 6(iii), one can verify that both the individual utilities for full-timers $u_+^*$ and for part-timers $\tilde{u}_+^*$ in the C$^+$M equilibrium are decreasing in both full-timers' labor pool size $M_f$ and part-timers' pool size $M_a$. Since $M_f = \gamma M$ for some constant $\gamma$ and that workers' utilities in the CM equilibrium $u^*$ and $u_+^*$ are constant of $M$, the ratio (OA.20) must decrease in the aggregate labor pool size $M$. □

**Proof of Proposition 7(iii)** We focus on the case with $c_a > (r_0 - \beta_{\mathsf{l}}c_f)/\beta_{\mathsf{h}}$. In Lemma OA.5 we specify an equivalent format of the company's optimization problem in the CM. Given that part-timers' average wait times now become $W_{a,t}^\pi = 1/((\mu_t - q_\pi \cdot q_{f,t}^\pi \lambda_f)(\mu_t - q_\pi \cdot q_{f,t}^\pi \lambda_f - q_{a,t}^\pi \lambda_{a,t})/\mu_t)$ (see Section 5.2), we can similarly derive the company's optimization problem in the C$^\pi$M equilibrium as follows using a similar argument as in the Proof of Lemma OA.5.

$$\max\{\max_{(q_{a,\mathsf{h}},q_{a,\mathsf{l}}) \in [0,1]^2} \textstyle\sum_{t \in \{\mathsf{h},\mathsf{l}\}} \beta_t (p_t - c_a(\frac{1}{(\mu_t - \lambda_f)(\mu_t - \lambda_f - q_{a,t}\lambda_{a,t})/\mu_t} + \frac{1}{\tau}))(\lambda_f + q_{a,t}\lambda_{a,t}),$$

$$\max_{(q_{a,\mathsf{h}},q_{a,\mathsf{l}})\in[0,1]^2}\beta_{\mathsf{h}}(p_{\mathsf{h}}-c_a(\tfrac{1}{(\mu_{\mathsf{h}}-\lambda_f)(\mu_{\mathsf{h}}-\lambda_f-q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}})/\mu_{\mathsf{h}}}+\tfrac{1}{\tau}))(\lambda_f+q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}})+\beta_{\mathsf{l}}(p_{\mathsf{l}}-c_f(\tfrac{1}{\mu_{\mathsf{l}}-q_{f,\mathsf{l}}\lambda_f}+\tfrac{1}{\tau}))q_{f,\mathsf{l}}\lambda_f,$$
$$\max_{(q_{f,\mathsf{h}},q_{a,\mathsf{l}})\in[0,1]^2}\beta_{\mathsf{h}}(p_{\mathsf{h}}-c_f(\tfrac{1}{\mu_{\mathsf{h}}-q_{f,\mathsf{h}}\lambda_f}+\tfrac{1}{\tau}))q_{f,\mathsf{h}}\lambda_f+\beta_{\mathsf{l}}(p_{\mathsf{l}}-c_a(\tfrac{1}{(\mu_{\mathsf{l}}-\lambda_f)(\mu_{\mathsf{l}}-\lambda_f-q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}})/\mu_{\mathsf{l}}}+\tfrac{1}{\tau}))(\lambda_f+q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}),$$
$$\max_{(q,q_{f,\mathsf{l}})\in[0,1]^2}\beta_{\mathsf{h}}(p_{\mathsf{h}}-\tfrac{r_0-\beta_{\mathsf{l}}c_f}{\beta_{\mathsf{h}}}(\tfrac{1}{\mu_{\mathsf{h}}-q\lambda_f}+\tfrac{1}{\tau}))q\lambda_f+\beta_{\mathsf{l}}(p_{\mathsf{l}}-c_f(\tfrac{1}{\mu_{\mathsf{l}}-q_{f,\mathsf{l}}\cdot q\lambda_f}+\tfrac{1}{\tau}))q_{f,\mathsf{l}}\cdot q\lambda_f\}. \quad \text{(OA.21)}$$

Following similar arguments as in the Proof of Proposition 1, one can verify that there exists $\underline{\mathsf{c}}$ such that if $c_a<\underline{\mathsf{c}}$, the average earning rates in the CM equilibrium $r^*_{\mathsf{h}}=r^*_{\mathsf{l}}=c_a$. Then full-timers' welfare in the CM will be $S^*=M_f(c_a-c_f)$ and part-timers' welfare $\tilde{S}^*=0$.

Analogously, there exists $\underline{\mathsf{c}}_\pi$ such that when $c_a<\underline{\mathsf{c}}_\pi$, solving the program (OA.21) will yield

$$\Pi^*_\pi=\max_{(q_{a,\mathsf{h}},q_{a,\mathsf{l}})\in[0,1]^2}\sum_{t\in\{\mathsf{h},\mathsf{l}\}}\beta_t(p_t-c_a(\frac{1}{(\mu_t-\lambda_f)(\mu_t-\lambda_f-q_{a,t}\lambda_{a,t})/\mu_t}+\frac{1}{\tau}))(\lambda_f+q_{a,t}\lambda_{a,t}).$$

Therefore, full-timers' average earning rates in the C$^\pi$M equilibrium will be

$$r^{\pi,*}_{f,t}=\frac{c_a(\frac{1}{(\mu_{\mathsf{h}}-\lambda_f)(\mu_{\mathsf{h}}-\lambda_f-q^*_{a,\mathsf{h}}\lambda_{a,\mathsf{h}})/\mu_{\mathsf{h}}}+\frac{1}{\tau})}{\frac{1}{\mu_{\mathsf{h}}-\lambda_f}+\frac{1}{\tau}}>c_a,\ t\in\{\mathsf{h},\mathsf{l}\},$$

and part-timers' average earning rates are

$$r^{\pi,*}_{a,t}=\frac{c_a(\frac{1}{(\mu_{\mathsf{h}}-\lambda_f)(\mu_{\mathsf{h}}-\lambda_f-q^*_{a,\mathsf{h}}\lambda_{a,\mathsf{h}})/\mu_{\mathsf{h}}}+\frac{1}{\tau})}{\frac{1}{(\mu_{\mathsf{h}}-\lambda_f)(\mu_{\mathsf{h}}-\lambda_f-q^*_{a,\mathsf{h}}\lambda_{a,\mathsf{h}})/\mu_{\mathsf{h}}}+\frac{1}{\tau}}=c_a,\ t\in\{\mathsf{h},\mathsf{l}\}.$$

Hence, when $c_a>(r_0-\beta_{\mathsf{l}}c_f)/\beta_{\mathsf{h}}$ and $c_a<\min\{\underline{\mathsf{c}},\underline{\mathsf{c}}_\pi\}$, we have $S^*_\pi=M_f(\sum_{t\in\{\mathsf{h},\mathsf{l}\}}\beta_t(r^{\pi,*}_{f,t}-c_f)^+-u_0)>S^*$ and $\tilde{S}^*_\pi=\tilde{S}^*=0$, and we obtain $\mathtt{S}^*_\pi>\mathtt{S}^*$. □

**Proof of Proposition 8**

**Proof of Proposition 8(i)** When $B=0$, according to our definition full-timers' welfare in the E$^d$M equilibrium $S^*_{E^d}=0\le S^*$. Since $S^*_{E^d}$ is continuous in $B$, there exists $\underline{B}_{E^d}\ge 0$ such that if $B\le\underline{B}_{E^d}$ we have $S^*_{E^d}\le S^*$. On the other hand, when $B\ge(\max_{t\in\{\mathsf{h},\mathsf{l}\}}p_t/(1/\mu_t+1/\tau)-r_0)/\alpha$, Proof of Lemma OA.11 implies that $\lambda^*_{E^d,\mathsf{h}}=\lambda^*_{E^d,\mathsf{l}}=0$. Hence we again have $S^*_{E^d}=0\le S^*$, and by the continuity of $S^*_{E^d}$ there exists $\overline{B}_{E^d}\le\max_{t\in\{\mathsf{h},\mathsf{l}\}}p_t/(1/\mu_t+1/\tau)-r_0$ such that if $B\ge\overline{B}_{E^d}$ we have $S^*_{E^d}\le S^*$. □

**Proof of Proposition 8(ii)** First note that, the company's optimal profit $\Pi^*_{E^d}$ in the E$^d$M is clearly less than the following,

$$\max_{(\lambda_{E^d,\mathsf{h}},\lambda_{E^d,\mathsf{l}})\in[0,\lambda_f]^2}\beta_{\mathsf{h}}\ (p_{\mathsf{h}}-r_{\alpha B}(\frac{1}{\mu_{\mathsf{h}}-\lambda_{E^d,\mathsf{h}}}+\frac{1}{\tau}))(\lambda_{E^d,\mathsf{h}}-\lambda_{E^d,\mathsf{l}})+\sum_{t\in\{\mathsf{h},\mathsf{l}\}}\beta_t(p_t-r_{\alpha B}(\frac{1}{\mu_t-\lambda_{E^d,t}}+\frac{1}{\tau}))\lambda_{E^d,\mathsf{l}}$$
$$=\sum_{t\in\{\mathsf{h},\mathsf{l}\}}\beta_t\ (p_t-r_{\alpha B}(\frac{1}{\mu_t-\lambda_{E^d,t}}+\frac{1}{\tau}))\lambda_{E^d,t}, \quad \text{(OA.22)}$$

since $r_B\ge r_{\alpha B}$.

Next, recall that $\lambda^*_{E^d,\mathsf{h}}$ and $\lambda^*_{E^d,\mathsf{l}}$ denote the optimal number of employees during peak and off-peak periods in the E$^d$M respectively. Since we have assumed that only full-timers can be hired as employees in the E$^d$M, $\lambda^*_{E^d,\mathsf{h}}$ and $\lambda^*_{E^d,\mathsf{l}}$ are independent of part-timers' availability parameters $\gamma_{a,\mathsf{h}}$ and $\gamma_{a,\mathsf{l}}$. We define the threshold $\overline{\gamma}_a$ as the value of $\gamma_{a,\mathsf{l}}$ that enforces the equality $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} = (1-2\gamma_{a,\mathsf{l}})M_a\tau = \lambda^*_{E^d,\mathsf{h}} - \lambda^*_{E^d,\mathsf{l}}$, or equivalently, $\overline{\gamma}_a \equiv 1/2 - (\lambda^*_{E^d,\mathsf{h}} - \lambda^*_{E^d,\mathsf{l}})/(2M_a\tau)$. That $\gamma_{a,\mathsf{l}} \le \overline{\gamma}_a$ implies $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \ge \lambda^*_{E^d,\mathsf{h}} - \lambda^*_{E^d,\mathsf{l}}$. One can verify that $\Pi^*$ is higher than (OA.22) and is thus also higher than $\Pi^*_{E^d}$ when $r_0 > c_a$ and $\lambda_{a,\mathsf{h}} - \lambda_{a,\mathsf{l}} \ge \lambda^*_{E^d,\mathsf{h}} - \lambda^*_{E^d,\mathsf{l}}$ by replacing $B$ with $\alpha B$ in Part II of the Proof of Theorem 4. $\square$

## OA5. Auxiliary Results

Here we establish some results in the contractor mode (CM) and the employee mode (EM) that will be useful for the proofs for our main findings. Proofs can be found in the online supplement.

### OA5.1 Implications of Lemma 1

LEMMA OA.1. *If for some piece-rate wages $(w_{\mathsf{h}}, w_{\mathsf{l}})$ the corresponding market equilibrium is $(\mathcal{Q}, \mathcal{R})$, then we have*

$$w_t = \hat{w}_t \equiv r_t\left(\frac{1}{\mu_t - q_{f,t}q\lambda_f - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}\right),\ \forall t \in \{\mathsf{h}, \mathsf{l}\}.$$

*Proof of Lemma OA.1.* Suppose instead there exists some $t \in \{\mathsf{h}, \mathsf{l}\}$ such that $w_t \neq \hat{w}_t$. Suppose first $w_{\mathsf{h}} > \hat{w}_{\mathsf{h}}$. Since $\mathcal{Q} \equiv (q, q_{f,\mathsf{h}}, q_{f,\mathsf{l}}, q_{a,\mathsf{h}}, q_{a,\mathsf{l}})$ is what has been induced by $(w_{\mathsf{h}}, w_{\mathsf{l}})$, by definition, the period-$\mathsf{h}$ average earning rate $r_{\mathsf{h}}$ shall be

$$r_{\mathsf{h}} = \frac{w_{\mathsf{h}}}{\frac{1}{\mu_{\mathsf{h}} - q_{f,\mathsf{h}}q\lambda_f - q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}} + \frac{1}{\tau}} > \frac{\hat{w}_{\mathsf{h}}}{\frac{1}{\mu_{\mathsf{h}} - q_{f,\mathsf{h}}q\lambda_f - q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}} + \frac{1}{\tau}} = r_{\mathsf{h}}.$$

Contradiction. Similarly one can obtain such a contradiction for $w_{\mathsf{h}} < \hat{w}_{\mathsf{h}}$ and for $w_{\mathsf{l}} \neq \hat{w}_{\mathsf{l}}$. Therefore, we must have $w_t = \hat{w}_t$ for both $t \in \{\mathsf{h}, \mathsf{l}\}$. $\square$

LEMMA OA.2. *The company's optimization problem (1) in the CM is equivalent to the following problem*

$$\max_{\mathcal{Q},\mathcal{R}}\ \Pi = \sum_{t\in\{h,l\}} \beta_t\left(p_t - r_t\left(\frac{1}{\mu_t - (q_{f,t}q\lambda_f + q_{a,t}\lambda_{a,t})} + \frac{1}{\tau}\right)\right)(q_{f,t}q\lambda_f + q_{a,t}\lambda_{a,t}) \tag{OA.23}$$

$$s.t.\quad q \in \mathfrak{q}(r_h, r_l), q_{i,t} \in \mathfrak{q}_i(r_t), \forall\ i \in \{f, a\},\ t \in \{h, l\}, \tag{OA.24}$$

*where*

$$\mathfrak{q}(r_h, r_l) = \begin{cases} \{1\} & \text{if } u > u_0 \\ \{0\} & \text{if } u < u_0 \\ [0,1] & \text{otherwise} \end{cases},\quad \mathfrak{q}_i(r_t) = \begin{cases} \{1\} & \text{if } r_t > c_i \\ \{0\} & \text{if } r_t < c_i \\ [0,1] & \text{otherwise} \end{cases}, \forall\ i \in \{f, a\},\ t \in \{h, l\},$$

*and $u \equiv \sum_{t\in\{h,l\}} \beta_t(r_t - c_f)^+$ and $u_0 \equiv r_0 - c_f$.*

*Proof of Lemma OA.2.* Denote by $\Pi^*$ the optimal value to problem (1), by $\Pi^\star$ the optimal value to problem (OA.23), and by $(\mathcal{Q}^\star, \mathcal{R}^\star)$ the solution to problem (OA.23). First, recall that $(w_{\mathsf{h}}^*, w_{\mathsf{l}}^*)$ denotes the solution to problem (1), and $(\mathcal{Q}^*, \mathcal{R}^*)$ denotes the corresponding market equilibrium in the CM. By the Proof of Lemma 1, $(\mathcal{Q}^*, \mathcal{R}^*)$ must be a feasible solution to problem (OA.23) as they meet the constraints (OA.24). Therefore, we have $\Pi^* \leq \Pi^\star$.

On the other hand, Lemma OA.1 implies the existence of $(w_{\mathsf{h}}^\star, w_{\mathsf{l}}^\star)$ that induce $(\mathcal{Q}^\star, \mathcal{R}^\star)$ as the market equilibrium for the CM. Since clearly $(w_{\mathsf{h}}^\star, w_{\mathsf{l}}^\star)$ are feasible to problem (1), we must have $\Pi^* \geq \Pi^\star$. As such, we have $\Pi^* = \Pi^\star$. $\square$

Corollary OA.1. *Define*

$$r_{f,t}(q_{f,t}) = \frac{w_t}{\frac{1}{\mu_t - q_{f,t}\cdot q\lambda_f - \mathbb{I}_{[c_f > c_a]}\lambda_{a,t}} + \frac{1}{\tau}}, r_{a,t}(q_{a,t}) = \frac{w_t}{\frac{1}{\mu_t - \mathbb{I}_{[c_f < c_a]}\cdot q\lambda_f - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}}, t \in \{\mathsf{h}, \mathsf{l}\},$$

*where $\mathbb{I}_{[\cdot]}$ is the indicator function. For any piece-rate wages $(w_\mathsf{h}, w_\mathsf{l})$ and full-timers' commitment probability $q$, in any period $t$ and for any group $i \in \{f, a\}$ of workers, their participation probability is (i) $q_{i,t} = 1$ if $r_{i,t}(1) \geq c_i$, (ii) $q_{i,t} = 0$ if $r_{i,t}(0) \leq c_i$, and (iii) $q_{i,t} \in (0,1)$ and is determined by the equation $r_{i,t}(q_{i,t}) = c_i$ if $c_i \in (r_{i,t}(1), r_{i,t}(0))$.*

*Proof of Corollary OA.1.* See the discussions in the proof of Lemma 1. □

Corollary OA.2. *For any piece-rate wages $(w_\mathsf{h}, w_\mathsf{l})$ and full-timers' commitment probability $q$, in any period $t$, if full-timers' opportunity cost is lower than that for part-timers (i.e., $c_f < c_a$), full-timers' participation probability will be strictly higher than part-timers' participation probability, i.e., $q_{f,t} > q_{a,t}$, unless $q_{a,t} = q_{f,t} = 0$ or 1; and vice versa.*

*Proof of Corollary OA.2.* See the discussions in the proof of Lemma 1. □

Corollary OA.3. *For any piece-rate wages $(w_\mathsf{h}, w_\mathsf{l})$ and full-timers' commitment probability $q$, in any period $t \in \{\mathsf{h}, \mathsf{l}\}$, full-timers' effective arrival rate (or their labor supply) $q_{f,t} \cdot q\lambda_f$ increases if their labor pool size $M_f$ increases or their opportunity cost $c_f$ decreases. Similarly, part-timers' effective arrival rate (or their labor supply) $q_{a,t}\lambda_{a,t}$ increases if their labor pool size $M_a$ increases or their opportunity cost $c_a$ decreases.*

*Proof of Corollary OA.3.* We will verify the results for part-timers. The results for full-timers can be verified similarly. First, Proof of Lemma 1 implies that part-timers' participation probability $q_{a,t}$ in any period $t \in \{\mathsf{h}, \mathsf{l}\}$ is indeed increasing as their opportunity cost $c_a$ decreases. Second, according to the derivation in the proof of Lemma 1, part-timers' effective arrival rate (or their labor supply) $q_{a,t}\lambda_{a,t}$ in any period $t$ is

$$q_{a,t}\lambda_{a,t} = \begin{cases} \lambda_{a,t} & \text{if } q \leq \hat{q}_{a,t}, \\ \max\{\mu_t - 1/(w_t/c_f - 1/\tau), 0\} & \text{otherwise} \end{cases}.$$

According to our definition in the proof of Lemma 1, we have $\hat{q}_{a,t} = (\mu_t - 1/(w_t/c_a - 1/\tau))/\lambda_f$ and is constant in $\lambda_{a,t}$. As such, once can verify that part-timers' effective arrival rate (or their labor supply) $q_{a,t}\lambda_{a,t}$ in any period $t$ is indeed increasing in part-timers' gross arrival rate $\lambda_{a,t}$, or equivalently, in part-timers' "labor pool size" $M_a$. □

Corollary OA.4. *Given full-timers' commitment probability $q$, the average earning rate $r_t$ in any period $t \in \{\mathsf{h}, \mathsf{l}\}$ is increasing in the piece-rate wages.*

*Proof of Corollary OA.4.* See the discussions in the proof of Lemma 1. ☐

COROLLARY OA.5. *Every component in the participation equilibrium* $\mathcal{Q} = (q, q_{f,h}, q_{f,l}, q_{a,h}, q_{a,l})$ *is increasing in the piece-rate wage* $w_t$ *for any* $t \in \{h, l\}$.

*Proof of Corollary OA.5.* See the discussions in the proof of Lemma 1. ☐

COROLLARY OA.6. *The transaction volume* $\lambda$ *in the CM is increasing in the piece-rate wage* $w_t$ *for any* $t \in \{h, l\}$.

*Proof of Corollary OA.6.* See the discussions in the proof of Lemma 1. ☐

## OA5.2 On the Company's Operations in the Contractor Mode

LEMMA OA.3. *For* $c_a \in (r_0, (r_0 - \beta_h c_f)/\beta_l)$, *the company's optimal profit* $\Pi^*$ *in the CM must be*

$$\begin{aligned}
\max\{&\max_{(q_{a,h},q_{a,l})\in[0,1]^2} \textstyle\sum_{t\in\{h,l\}} \beta_t(p_t - c_a(\frac{1}{\mu_t-\lambda_f-q_{a,t}\lambda_{a,t}}+\frac{1}{\tau}))(\lambda_f+q_{a,t}\lambda_{a,t}),\\
&\max_{(q,q_{a,l})\in[0,1]^2} \beta_h(p_h - \tfrac{r_0-\beta_l c_a}{\beta_h}(\tfrac{1}{\mu_h-q\lambda_f}+\tfrac{1}{\tau}))q\lambda_f + \beta_l(p_l - c_a(\tfrac{1}{\mu_l-q\lambda_f-q_{a,l}\lambda_{a,l}}+\tfrac{1}{\tau}))(q\lambda_f+q_{a,l}\lambda_{a,l}),\\
&\max_{(q,q_{a,h})\in[0,1]^2} \beta_h(p_h - c_a(\tfrac{1}{\mu_h-q\lambda_f-q_{a,h}\lambda_{a,h}}+\tfrac{1}{\tau}))(q\lambda_f+q_{a,h}\lambda_{a,h}) + \beta_l(p_l - \tfrac{r_0-\beta_h c_a}{\beta_l}(\tfrac{1}{\mu_l-q\lambda_f}+\tfrac{1}{\tau}))q\lambda_f,\\
&\max_{r_h\in(c_a,\frac{r_0-\beta_l c_f}{\beta_h}),q\in[0,1]} \beta_h(p_h - r_h(\tfrac{1}{\mu_h-q\lambda_f-\lambda_{a,h}}+\tfrac{1}{\tau}))(q\lambda_f+\lambda_{a,h}) + \beta_l(p_l - \tfrac{r_0-\beta_h r_h}{\beta_l}(\tfrac{1}{\mu_l-q\lambda_f}+\tfrac{1}{\tau}))q\lambda_f,\\
&\max_{(q,q_{f,l})\in[0,1]^2} \beta_h(p_h - \tfrac{r_0-\beta_l c_f}{\beta_h}(\tfrac{1}{\mu_h-q\lambda_f-\lambda_{a,h}}+\tfrac{1}{\tau}))(q\lambda_f+\lambda_{a,h}) + \beta_l(p_l - c_f(\tfrac{1}{\mu_l-q_{f,l}\cdot q\lambda_f}+\tfrac{1}{\tau}))q_{f,l}\cdot q\lambda_f\}.
\end{aligned}$$

*Proof of Lemma OA.3.* Lemma OA.2 shows that the company's problem in the CM is to optimize over the earning rate schedule $(r_{\mathsf{h}}, r_{\mathsf{l}})$, full-timers' commitment probability, their and part-timers' participation probabilities $(q, q_{f,\mathsf{h}}, q_{f,\mathsf{l}}, q_{a,\mathsf{h}}, q_{a,\mathsf{l}})$. The key to the proof is to pin down the optimal average earning rates $(r^*_{\mathsf{h}}, r^*_{\mathsf{l}})$. The company's optimal control with respect to workers' participations will then become straightforward according to Lemma OA.2. Note that, at optimality, we either have $r^*_{\mathsf{h}} = r^*_{\mathsf{l}}$ or $r^*_{\mathsf{h}} \neq r^*_{\mathsf{l}}$. We will explore optimal earning rates falling into either batch. We will highlight earning schedules that can be verified as suboptimal directly with a *Remark*, and will also summarize all potentially optimal earning schedules at the end of the whole discussion.

Start with $r_{\mathsf{h}} = r_{\mathsf{l}} = r$ for some $r$. We claim that there are only two values of $r$ can possibly be optimal: $r = r_0$ and $r = c_a$. For any $r < r_0$, no full-timer will commit and no part-timer will participate. For $r \in (r_0, c_a)$, all full-timers will commit (and participate) but no part-timer will participate. Yet then such a $r$ yields a lower profit than $r_0$, which also attracts all full-timers but no part-timer. Finally, all $r > c_a$ yield a lower profit than $c_a$, which is enough to all full-timers and part-timers.

Now we turn to $r_{\mathsf{h}} \neq r_{\mathsf{l}}$. First, it is never optimal to have $r_t < c_f$ for any $t \in \{\mathsf{h}, \mathsf{l}\}$ because no worker will be incentivized in that period. The company can make a (weakly) higher profit by setting $r_t = c_f$ and enrolling at least some committed full-timers. As such, we start with $r_{\mathsf{h}} = c_f$.

• $r_{\mathsf{h}} = c_f$. The only possibly value of $r_{\mathsf{l}}$ for optimality is $(r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$. Note that because $\beta_{\mathsf{h}} < \beta_{\mathsf{l}}$, we have $(r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}} > (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$. For $r_{\mathsf{l}} > (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}} > c_a$, all part-timers will participate and all full-timers will commit because they all have a positive surplus (i.e., $u > u_0$). As such, the company can clearly make a higher profit by strictly lowering $r_{\mathsf{l}}$ by some $\epsilon > 0$. For $r_{\mathsf{l}} \in (c_a, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$, no full-timer will commit because $u < u_0$ and such an $r_{\mathsf{l}}$ will be too high to enroll all part-timers. For $r_{\mathsf{l}} < c_a$, no full- or part-timer will ever participate. For $r_{\mathsf{l}} = c_a$, no full-timer will commit and only some part-timers will participate during off-peak periods; that is, the company makes zero profit in peak periods. The company can then raise $r_{\mathsf{h}}$ to $(r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}$ so that it is incentive compatible for full-timers to commit (i.e., $u = u_0$) and make a higher profit by enrolling some full-timers in both periods.

Given that $(r_{\mathsf{h}}, r_{\mathsf{l}}) = (c_f, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$, the company's optimization problem is

$$\max_{q, q_{f,\mathsf{h}} \in [0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_f(\frac{1}{\mu_{\mathsf{h}} - q_{f,\mathsf{h}} \cdot q\lambda_f} + \frac{1}{\tau}))q_{f,\mathsf{h}} \cdot q\lambda_f + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f - \lambda_{a,\mathsf{l}}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{l}}).$$

We claim that the optimal participation probability among committed full-timers in peak period $q^*_{f,\mathsf{h}}$ must equal 1. To verify this, denote the profit value in the optimization program above by $\hat{\Pi}$ and the optimal commitment probability among full-timers by $\hat{q}^*$. Solving the optimization program yields $q^*_{f,\mathsf{h}} = \min\{1, (\mu_{\mathsf{h}} - \sqrt{\mu_{\mathsf{h}} \tau c_f/(p_{\mathsf{h}}\tau - c_f)})/\hat{q}^*\lambda_f\}$. Suppose instead $q^*_{f,\mathsf{h}} < 1$, which implies that $q^*_{f,\mathsf{h}} = (\mu_{\mathsf{h}} - \sqrt{\mu_{\mathsf{h}} \tau c_f/(p_{\mathsf{h}}\tau - c_f)})/\hat{q}^*\lambda_f$ and $\hat{q}^*\lambda_f > \mu_{\mathsf{h}} - \sqrt{\mu_{\mathsf{h}} \tau c_f/(p_{\mathsf{h}}\tau - c_f)}$. But then we have

$$\frac{d\hat{\Pi}}{dq}_{q=\hat{q}^*} = \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_f(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \hat{q}^*\lambda_f)^2} + \frac{1}{\tau})) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - \hat{q}^*\lambda_f - \lambda_{a,\mathsf{l}})^2} + \frac{1}{\tau})) < 0$$

because (a) the root to the second summand equals $\mu_{\mathsf{l}} - \sqrt{\mu_{\mathsf{l}} \tau (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}/(p_{\mathsf{h}}\tau - (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})} - \lambda_{a,\mathsf{l}} < \mu_{\mathsf{h}} - \sqrt{\mu_{\mathsf{h}} \tau c_f/(p_{\mathsf{h}}\tau - c_f)} < \hat{q}^*\lambda_f$ and (b) the profit function $\hat{\Pi}$ is strictly concave. The inequality implies that $q = \hat{q}^*$ is strictly suboptimal. Contradiction.

• $r_{\mathsf{h}} \in (c_f, (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}]$. The only possibly value of $r_{\mathsf{l}}$ for optimality is $(r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}})/\beta_{\mathsf{l}}$. For $r_{\mathsf{l}} < (r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}})/\beta_{\mathsf{l}}$, it is not incentive compatible for any full-timer to commit, and following the argument in the $r_{\mathsf{h}} = c_f$ case one can see that such $(r_{\mathsf{h}}, r_{\mathsf{l}})$ cannot be optimal. For $r_{\mathsf{l}} > (r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}})/\beta_{\mathsf{l}}$, it will be more than incentive compatible for full-timers to commit (i.e., $u > u_0$) and notice that $r_{\mathsf{l}}$ will be strictly higher than $c_a$ since $r_{\mathsf{h}} \leq (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}$; as such, the company can strictly lower $r_{\mathsf{l}}$ to enroll part-timers and make full-timers commit.

Now, with $r_{\mathsf{h}} \in (c_f, (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}]$ and $r_{\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}})/\beta_{\mathsf{l}}$, the company's optimization problem is

$$\max\{\max_{r_{\mathsf{h}} < \frac{r_0 - \beta_{\mathsf{l}} c_a}{\beta_{\mathsf{h}}}, q \in [0,1]} \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}}}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f - \lambda_{a,\mathsf{l}}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{l}}),$$

$$\max_{q,q_{a,\mathsf{l}}\in[0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - \frac{r_0 - \beta_{\mathsf{l}} c_a}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f - q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}} + \frac{1}{\tau}))(q\lambda_f + q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}})\}.$$

To understand the optimal solutions to the problem above, fix $r_{\mathsf{h}} < (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}$ and let the company only optimizes full-timers' commitment probability $q$. Denote the optimal probability by $\hat{q}^*$ and the corresponding optimal profit by $\hat{\Pi}^*$. According to the Envelop Theorem, we have

$$\frac{d\hat{\Pi}^*}{dr_{\mathsf{h}}} = -\beta_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - \hat{q}^*\lambda_f} + \frac{1}{\tau})\hat{q}^*\lambda_f + \beta_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{l}} - \hat{q}^*\lambda_f - \lambda_{a,\mathsf{l}}} + \frac{1}{\tau})(\hat{q}^*\lambda_f + \lambda_{a,\mathsf{l}}) > 0.$$

because $\mu_{\mathsf{h}} > \mu_{\mathsf{l}}$. This implies that the optimal earning rate in peak periods $r_{\mathsf{h}}^*$ must equal $(r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}$.

Combining the discussion in this case and that in the previous case (i.e., $r_{\mathsf{h}} = c_f$), one can also see that $r_{\mathsf{h}} = c_f$ will not qualify for optimality.

• $r_{\mathsf{h}} \in ((r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}, c_a]$. Following the argument in the previous case one can similarly verify that the only possible value of $r_{\mathsf{l}}$ for optimality is again $(r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$. Because now $r_{\mathsf{h}} > (r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}$, we have $r_{\mathsf{l}} < c_a$. The company's optimization problem is then

$$\begin{aligned}\max\{ &\max_{r_{\mathsf{h}} < c_a, q\in[0,1]} \beta_{\mathsf{h}}\ (p_{\mathsf{h}} - r_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}}}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f, \\ &\max_{q,q_{a,\mathsf{h}}\in[0,1]^2} \beta_{\mathsf{h}}\ (p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f - q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))(q\lambda_f + q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_a}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f\}.\end{aligned}$$

Similarly as in the previous case, one can verify that the optimal earning rate in peak periods $r_{\mathsf{h}}^*$ must equal $c_a$. Note that, this result rules out the possibility for $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ to be optimal because $r_0 \in ((r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}, c_a)$.

• $r_{\mathsf{h}} \in (c_a, (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}]$. Following the argument in earlier cases one can similarly verify that the only possible value of $r_{\mathsf{l}}$ for optimality is again $(r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$. The company's optimization problem is then

$$\begin{aligned}\max\{ &\max_{r_{\mathsf{h}} < \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}, q\in[0,1]} \beta_{\mathsf{h}}\ (p_{\mathsf{h}} - r_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}}}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f} + \frac{1}{\tau}))q\lambda_f, \\ &\max_{q,q_{f,\mathsf{l}}\in[0,1]^2} \beta_{\mathsf{h}}\ (p_{\mathsf{h}} - \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}(\frac{1}{\mu_{\mathsf{h}} - q\lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_f(\frac{1}{\mu_{\mathsf{l}} - q_{f,\mathsf{l}} \cdot q\lambda_f} + \frac{1}{\tau}))q_{f,\mathsf{l}} \cdot q\lambda_f\}.\end{aligned}$$

Note that unlike the previous case, any $r_{\mathsf{h}} \in (c_a, (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}]$ can potentially be optimal. To see this, fix $r_{\mathsf{h}} < (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$ and let the company only optimizes full-timers' commitment probability $q$. Denote the optimal probability by $\hat{q}^*$ and the corresponding optimal profit by $\hat{\Pi}^*$. According to the Envelop Theorem, we have

$$\frac{d\hat{\Pi}^*}{dr_{\mathsf{h}}} = -\beta_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - \hat{q}^*\lambda_f - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau})(\hat{q}^*\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{l}} - \hat{q}^*\lambda_f} + \frac{1}{\tau})\hat{q}^*\lambda_f,$$

which can either be positive or negative (depending on, e.g., the magnitude of $\lambda_{a,\mathsf{h}}$).

To summarize, when $c_a \in (r_0, (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}})$, we find that only the following earning rate schedules can possible be optimal: (a) $r_{\mathsf{h}} = r_{\mathsf{l}} = c_a$, (b) $r_{\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}})/r_{\mathsf{l}}$ and $r_{\mathsf{h}}$ takes value from one of the following: (b.1) $(r_0 - \beta_{\mathsf{l}} c_a)/\beta_{\mathsf{h}}$ and (b.2) $[c_a, (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}]$. The company's optimal control with respect to workers' participations then follow the prescriptions in Lemma OA.2 for any particular earning rate schedule $(r_{\mathsf{h}}, r_{\mathsf{l}})$. □

Lemma OA.4. *For $c_a < c_f$, the company's optimal profit $\Pi^*$ in the CM must be*

$$\begin{aligned}
&\max\{\max_{(q_{a,h},q_{a,l})\in[0,1]^2} \textstyle\sum_{t\in\{h,l\}} \beta_t(p_t - c_a(\frac{1}{\mu_t - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}))q_{a,t}\lambda_{a,t},\\
&\max_{q,q_{a,h}\in[0,1]^2} \beta_h(p_h - c_a(\tfrac{1}{\mu_h - q_{a,h}\lambda_{a,h}} + \tfrac{1}{\tau}))q_{a,h}\lambda_{a,h} + \beta_l(p_l - \tfrac{r_0 - \beta_h c_f}{\beta_l}(\tfrac{1}{\mu_l - q\lambda_f - \lambda_{a,l}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,l}),\\
&\max_{q,q_{f,h}\in[0,1]^2} \beta_h(p_h - c_f(\tfrac{1}{\mu_h - q_{f,h}q\lambda_f - \lambda_{a,h}} + \tfrac{1}{\tau}))(q_{f,h}q\lambda_f + \lambda_{a,h}) + \beta_l(p_l - \tfrac{r_0 - \beta_h c_f}{\beta_l}(\tfrac{1}{\mu_l - q\lambda_f - \lambda_{a,l}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,l}),\\
&\max_{r_h\in(c_f,\frac{r_0 - \beta_l c_f}{\beta_h}),q\in[0,1]} \beta_h(p_h - r_h(\tfrac{1}{\mu_h - q\lambda_f - \lambda_{a,h}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,h}) + \beta_l(p_l - \tfrac{r_0 - \beta_h r_h}{\beta_l}(\tfrac{1}{\mu_l - q\lambda_f - \lambda_{a,l}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,l})\\
&\max_{q,q_{f,l}\in[0,1]^2} \beta_h(p_h - \tfrac{r_0 - \beta_l c_f}{\beta_h}(\tfrac{1}{\mu_h - q\lambda_f - \lambda_{a,h}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,h}) + \beta_l(p_l - c_f(\tfrac{1}{\mu_l - q_{f,l}q\lambda_f - \lambda_{a,l}} + \tfrac{1}{\tau}))(q_{f,l}q\lambda_f + \lambda_{a,l}),\\
&\max_{q\in[0,1]} \beta_h(p_h - \tfrac{r_0 - \beta_l c_f}{\beta_h}(\tfrac{1}{\mu_h - q\lambda_f - \lambda_{a,h}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,h}) + \beta_l(p_l - c_a(\tfrac{1}{\mu_l - q_{a,l}\lambda_{a,l}} + \tfrac{1}{\tau}))q_{a,l}\lambda_{a,l}\}.
\end{aligned}$$

*Proof of Lemma OA.4.* Same as in the Proof of Lemma OA.3, we start with $r_{\mathsf{h}} = r_{\mathsf{l}} = r$ for some $r$. Following a similar argument as in the Proof of Lemma OA.3, we can verify that again there are only two values of $r$ to be potentially optimal: $r = r_0$ and $r = c_a$.

Next we look for potentially optimal earning schedules $(r_{\mathsf{h}}, r_{\mathsf{l}})$ such that $r_{\mathsf{h}} \neq r_{\mathsf{l}}$. We start with $r_{\mathsf{h}} = c_a$ since any $r_{\mathsf{h}} < c_a$ will enroll no worker in the peak periods and thus cannot be optimal.

- $r_{\mathsf{h}} \in [c_a, c_f)$. The only possible value of $r_{\mathsf{l}}$ for optimality is $(r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$. For any $r_{\mathsf{l}} < (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$, since no full-timer will commit as the utility on the platform $u$ is strictly less than their outside option $u_0$, the company can reduce $r_{\mathsf{l}}$ to $c_a$ and only enroll part-timers, but this is already covered in the homogeneous scenario (i.e., $r_{\mathsf{h}} = r_{\mathsf{l}}$). For any $r_{\mathsf{l}} > (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$, both full-timers and part-timers are given more than enough incentives for them to participate in off-peak periods. The company's optimization problem is then

$$\begin{aligned}
\max\{&\max_{q,q_{a,\mathsf{h}}\in[0,1]^2} \beta_{\mathsf{h}}\ (p_{\mathsf{h}} - c_a(\frac{1}{\mu_{\mathsf{h}} - q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))q_{a,\mathsf{h}}\lambda_{a,\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f - \lambda_{a,\mathsf{l}}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{l}}),\\
&\max_{r_{\mathsf{h}}\in(c_a,c_f),q\in[0,1]} \beta_{\mathsf{h}}\ (p_{\mathsf{h}} - r_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau}))\lambda_{a,\mathsf{h}} + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0 - \beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f - \lambda_{a,\mathsf{l}}} + \frac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{l}})\}.
\end{aligned}$$

To understand the optimal solutions to the problem above, fix $r_{\mathsf{h}} > c_a$ and let the company only optimizes full-timers' commitment probability $q$. Denote the optimal probability by $\hat{q}^*$ and the corresponding optimal profit by $\hat{\Pi}^*$. According to the Envelop Theorem, we have

$$\frac{d\hat{\Pi}^*}{dr_{\mathsf{h}}} = -\beta_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}} - \lambda_{a,\mathsf{h}}} + \frac{1}{\tau})\lambda_{a,\mathsf{h}} < 0.$$

This implies that the optimal earning rate in peak periods $r_{\mathsf{h}}^*$ must equal $c_a$.

• $r_{\mathsf{h}} \in [c_f, (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}})$. Following a similar argument as in the previous case, we can verify that the only possible value of $r_{\mathsf{l}}$ for optimality is $(r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}})/\beta_{\mathsf{l}}$. The company's optimization problem is then

$$\max\{ \quad \max_{q,q_{f,\mathsf{h}}\in[0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_f(\tfrac{1}{\mu_{\mathsf{h}} - q_{f,\mathsf{h}} q\lambda_f - \lambda_{a,\mathsf{h}}} + \tfrac{1}{\tau}))(q_{f,\mathsf{h}} q\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \tfrac{r_0 - \beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\tfrac{1}{\mu_{\mathsf{l}} - q\lambda_f - \lambda_{a,\mathsf{l}}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{l}}),$$
$$\max_{r_{\mathsf{h}}\in(c_f, \frac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}), q\in[0,1]} \beta_{\mathsf{h}}(p_{\mathsf{h}} - r_{\mathsf{h}}(\tfrac{1}{\mu_{\mathsf{h}} - q\lambda_f - \lambda_{a,\mathsf{h}}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \tfrac{r_0 - \beta_{\mathsf{h}} r_{\mathsf{h}}}{\beta_{\mathsf{l}}}(\tfrac{1}{\mu_{\mathsf{l}} - q\lambda_f - \lambda_{a,\mathsf{l}}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{l}})\}.$$

Notice that because $r_0 \in (c_f, (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}})$, this includes the $r_{\mathsf{h}} = r_{\mathsf{l}} = r_0$ case in the homogeneous scenario (i.e., $r_{\mathsf{h}} = r_{\mathsf{l}}$).

• $r_{\mathsf{h}} = (r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}$. There are two possible values of $r_{\mathsf{l}}$ for optimality: $c_f$ and $c_a$. The company's optimization problem is then

$$\max\{ \max_{q,q_{f,\mathsf{l}}\in[0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - \tfrac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}(\tfrac{1}{\mu_{\mathsf{h}} - q\lambda_f - \lambda_{a,\mathsf{h}}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_f(\tfrac{1}{\mu_{\mathsf{l}} - q_{f,\mathsf{l}} q\lambda_f - \lambda_{a,\mathsf{l}}} + \tfrac{1}{\tau}))(q_{f,\mathsf{l}} q\lambda_f + \lambda_{a,\mathsf{l}}),$$
$$\max_{q\in[0,1]} \beta_{\mathsf{h}}(p_{\mathsf{h}} - \tfrac{r_0 - \beta_{\mathsf{l}} c_f}{\beta_{\mathsf{h}}}(\tfrac{1}{\mu_{\mathsf{h}} - q\lambda_f - \lambda_{a,\mathsf{h}}} + \tfrac{1}{\tau}))(q\lambda_f + \lambda_{a,\mathsf{h}}) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - c_a(\tfrac{1}{\mu_{\mathsf{l}} - q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}} + \tfrac{1}{\tau}))q_{a,\mathsf{l}}\lambda_{a,\mathsf{l}}\}.$$

□

Lemma OA.5. *For $c_a > (r_0 - \beta_l c_f)/\beta_h$, the company's optimal profit $\Pi^*$ in the CM must be*

$$\max\{ \max_{(q_{a,h}, q_{a,l})\in[0,1]^2} \sum_{t\in\{h,l\}} \beta_t(p_t - c_a(\frac{1}{\mu_t - \lambda_f - q_{a,t}\lambda_{a,t}} + \frac{1}{\tau}))(\lambda_f + q_{a,t}\lambda_{a,t}),$$
$$\max_{(q_{a,h}, q_{a,l})\in[0,1]^2} \beta_h(p_h - c_a(\frac{1}{\mu_h - \lambda_f - q_{a,h}\lambda_{a,h}} + \frac{1}{\tau}))(\lambda_f + q_{a,h}\lambda_{a,h}) + \beta_l(p_l - c_f(\frac{1}{\mu_l - q_{f,l}\lambda_f} + \frac{1}{\tau}))q_{f,l}\lambda_f,$$
$$\max_{(q_{f,h}, q_{a,l})\in[0,1]^2} \beta_h(p_h - c_f(\frac{1}{\mu_h - q_{f,h}\lambda_f} + \frac{1}{\tau}))q_{f,h}\lambda_f + \beta_l(p_l - c_a(\frac{1}{\mu_l - \lambda_f - q_{a,l}\lambda_{a,l}} + \frac{1}{\tau}))(\lambda_f + q_{a,l}\lambda_{a,l}),$$
$$\max_{(q, q_{f,l})\in[0,1]^2} \beta_h(p_h - \frac{r_0 - \beta_l c_f}{\beta_h}(\frac{1}{\mu_h - q\lambda_f} + \frac{1}{\tau}))q\lambda_f + \beta_l(p_l - c_f(\frac{1}{\mu_l - q_{f,l}\cdot q\lambda_f} + \frac{1}{\tau}))q_{f,l}\cdot q\lambda_f\}.$$

*Proof of Lemma OA.5.* Again the key to the proof is to find out all possibly optimal earning schedules $(r_{\mathsf{h}}^*, r_{\mathsf{l}}^*)$.

Suppose first the company wishes to attract only full-timers. The CM then essentially becomes a full-timer-only CM and, according to Lemma OA.6, the only possibly optimal earning schedule is $(r_{\mathsf{h}}^*, r_{\mathsf{l}}^*) = ((r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}, c_f)$. Notice that because $(r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}} < c_a$, indeed no part-timer will ever participate throughout the day.

Then suppose the company will enroll both full- and part-timers. Three possible cases: enrolling part-timers (a) only in peak periods, (b) only in off-peak periods, or (c) in both periods. For case (a), the optimal earning schedule must be $(r_{\mathsf{h}}^*, r_{\mathsf{l}}^*) = (c_a, c_f)$; any $r_{\mathsf{h}} > c_a$ is more than enough to incentivize part-timers and make full-timers commit (notice that full-timers' utility $u > u_0$ already at $r_{\mathsf{h}} = c_a$), and any $r_{\mathsf{l}} > c_f$ is too high to incentivize committed full-timers to participate. Then similarly for (b), the optimal earning schedule must be $(r^*_{\mathsf{h}}, r^*_{\mathsf{l}}) = (c_f, c_a)$. Finally for (c) the optimal schedule is clearly $(r^*_{\mathsf{h}}, r^*_{\mathsf{l}}) = (c_a, c_a)$. □

LEMMA OA.6. *The company's optimization problem in a full-timer-only CM is equivalent to*

$$\max_{(q,q_{f,l})\in[0,1]^2} \beta_h(p_h - \frac{r_0-\beta_l c_f}{\beta_h}(\frac{1}{\mu_h - q\lambda_f}+\frac{1}{\tau}))q\lambda_f + \beta_l(p_l - c_f(\frac{1}{\mu_l - q_{f,l}q\lambda_f}+\frac{1}{\tau}))q_{f,l}q\lambda_f. \quad \text{(OA.25)}$$

*Proof of Lemma OA.6.* Lemma OA.2 implies that in the full-timer-only CM, the company's problem is to optimize over the earning rate schedule $(r_{\mathsf{h}}, r_{\mathsf{l}})$ and full-timers' commitment and participation probabilities $(q, q_{f,\mathsf{h}}, q_{f,\mathsf{l}})$. The key to the proof is to show that the optimal average earning rates $(r^*_{\mathsf{h}}, r^*_{\mathsf{l}})$ must be $((r_0 - \beta_{\mathsf{l}} c_f)/\beta_{\mathsf{h}}, c_f)$. The company's control with respect to full-timers' commitment and participation probabilities will then become straightforward.

First, it is not optimal to have $r^*_t < c_f$ in any period $t \in \{\mathsf{h}, \mathsf{l}\}$, since then no committed full-timer (if any) will participate; the company can increase the profit by raising $r^*_t$ at least to $c_f$. Next, the optimal earning schedule $(r^*_{\mathsf{h}}, r^*_{\mathsf{l}})$ must satisfy $u^* = u_0$, or equivalently, that $\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (r^*_t - c_f)^+ = r_0 - c_f$. In other words, the company will not leave full-timers any positive surplus. Suppose the opposite is true, i.e., $u^* > u_0$. Lemma OA.8 implies that full-timers' joining probability $q^* = 1$ and there is at least one period $t \in \{\mathsf{h}, \mathsf{l}\}$ such that the earning rate $r^*_t$ is strictly higher than full-timers' opportunity cost $c_f$ and their participation probability $q^*_{f,t} = 1$. Then the company can clearly make a higher profit by lowering $r^*_t$ by some small amount $\epsilon > 0$ such that the same number of full-timers will commit and participate. Contradiction.

We thus focus on earning schedules $(r_{\mathsf{h}}, r_{\mathsf{l}})$ that satisfy $\sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (r_t - c_f)^+ = r_0 - c_f$ and $r_t \geq c_f$ for $t \in \{\mathsf{h}, \mathsf{l}\}$. Start with $r_{\mathsf{h}} = c_f$. We then have $r_{\mathsf{l}} = (r_0 - \beta_{\mathsf{h}} c_f)/\beta_{\mathsf{l}}$ and according to Lemma OA.2, the company's optimization problem is

$$\max_{q,q_{f,\mathsf{h}}\in[0,1]^2} \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_f(\frac{1}{\mu_{\mathsf{h}} - q_{f,\mathsf{h}}q\lambda_f}+\frac{1}{\tau}))q_{f,\mathsf{h}}q\lambda_f + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0-\beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}} - q\lambda_f}+\frac{1}{\tau}))q\lambda_f. \quad \text{(OA.26)}$$

Below we verify that the optimal $q^*_{f,\mathsf{h}}$ to (OA.26) equals 1.

Denote the profit value in (OA.26) by $\hat{\Pi}$ and the optimal commitment probability to (OA.26) by $\hat{q}^*$. Solving the optimization program yields $q^*_{f,\mathsf{h}} = \min\{1, (\mu_{\mathsf{h}} - \sqrt{\mu_{\mathsf{h}}\tau c_f/(p_{\mathsf{h}}\tau - c_f)})/\hat{q}^*\lambda_f\}$. Suppose instead $q^*_{f,\mathsf{h}} < 1$, which implies that $q^*_{f,\mathsf{h}} = (\mu_{\mathsf{h}} - \sqrt{\mu_{\mathsf{h}}\tau c_f/(p_{\mathsf{h}}\tau - c_f)})/\hat{q}^*\lambda_f$ and $\hat{q}^*\lambda_f > \mu_{\mathsf{h}} - \sqrt{\mu_{\mathsf{h}}\tau c_f/(p_{\mathsf{h}}\tau - c_f)}$. But then we have

$$\frac{d\hat{\Pi}}{dq}_{q=\hat{q}^*} = \beta_{\mathsf{h}}(p_{\mathsf{h}} - c_f(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \hat{q}^*\lambda_f)^2}+\frac{1}{\tau})) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - \frac{r_0-\beta_{\mathsf{h}} c_f}{\beta_{\mathsf{l}}}(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - \hat{q}^*\lambda_f)^2}+\frac{1}{\tau})) < 0$$

because (a) the root to the second summand equals $\mu_{\mathsf{l}}-\sqrt{\mu_{\mathsf{l}}\tau(r_0-\beta_{\mathsf{h}}c_f)/\beta_{\mathsf{l}}/(p_{\mathsf{h}}\tau-(r_0-\beta_{\mathsf{h}}c_f)/\beta_{\mathsf{l}})}<\mu_{\mathsf{h}}-\sqrt{\mu_{\mathsf{h}}\tau c_f/(p_{\mathsf{h}}\tau-c_f)}<\hat{q}^*\lambda_f$ and (b) the profit function $\hat{\Pi}$ is strictly concave. The inequality implies that $q=\hat{q}^*$ is strictly suboptimal. Contradiction.

Next, we check if any $r_{\mathsf{h}}>c_f$ will yield a higher profit than $r_{\mathsf{h}}=c_f$. For $r_{\mathsf{h}}\in(c_f,(r_0-\beta_{\mathsf{l}}c_f)/\beta_{\mathsf{h}})$, we have $r_{\mathsf{l}}=(r_0-\beta_{\mathsf{h}}r_{\mathsf{h}})/\beta_{\mathsf{l}}\in(c_f,(r_0-\beta_{\mathsf{h}}c_f)/\beta_{\mathsf{l}})$ and according to Lemma OA.2, the company's optimization problem is

$$\max_{q\in[0,1]}\beta_{\mathsf{h}}(p_{\mathsf{h}}-r_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}}-q\lambda_f}+\frac{1}{\tau}))q\lambda_f+\beta_{\mathsf{l}}(p_{\mathsf{l}}-\frac{r_0-\beta_{\mathsf{h}}r_{\mathsf{h}}}{\beta_{\mathsf{l}}}(\frac{1}{\mu_{\mathsf{l}}-q\lambda_f}+\frac{1}{\tau}))q\lambda_f.$$

Denote the optimal value by $\hat{\Pi}^*$ and the optimal solution by $\hat{q}^*$. Using the Envelop Theorem, we obtain

$$\begin{aligned}\frac{d\hat{\Pi}^*}{dr_{\mathsf{h}}}&=-\beta_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{h}}-\hat{q}^*\lambda_f}+\frac{1}{\tau})\hat{q}^*\lambda_f+\beta_{\mathsf{h}}(\frac{1}{\mu_{\mathsf{l}}-\hat{q}^*\lambda_f}+\frac{1}{\tau})\hat{q}^*\lambda_f\\&=\beta_{\mathsf{h}}(\frac{\hat{q}^*\lambda_f}{\mu_{\mathsf{l}}-\hat{q}^*\lambda_f}-\frac{\hat{q}^*\lambda_f}{\mu_{\mathsf{h}}-\hat{q}^*\lambda_f})>0.\end{aligned}$$

This implies that the optimal earning rate in peak periods $r_{\mathsf{h}}^*$ must be at least $(r_0-\beta_{\mathsf{l}}c_f)/\beta_{\mathsf{h}}$ (i.e., $r_{\mathsf{h}}^*\geq(r_0-\beta_{\mathsf{l}}c_f)/\beta_{\mathsf{h}}$). In fact, we must have $r_{\mathsf{h}}^*=(r_0-\beta_{\mathsf{l}}c_f)/\beta_{\mathsf{h}}$ as otherwise again $u^*>u_0$ and $r_{\mathsf{h}}^*$ is too high to be optimal. □

### OA5.3 Miscellaneous

Lemma OA.7. *If for some piece-rate wages $(w_h,w_l)$, full-timers' commitment probability $q=1$, there exists at least one $t\in\{h,l\}$ such that the induced average earning rate $r_t\geq r_0$ and the participation probability in that period $q_{f,t}=1$.*

*Proof of Lemma OA.7.* Suppose the induced earning rates in both periods are lower than $r_0$, i.e., $r_t<r_0$ for any $t$. Then the expected utility on the platform $u=\sum_{t\in\{\mathsf{h},\mathsf{l}\}}\beta_t(r_t-c_f)^+<u_0$, which implies that no full-timer will ever commit (i.e., $q=0$). Contradiction. So there must be some $t\in\{\mathsf{h},\mathsf{l}\}$ such that $r_t\geq r_0$. The result that full-timers' participation probability $q_{f,t}=1$ then immediately follows that $r_t>c_f$ given our assumption $r_0>c_f$. □

Lemma OA.8. *If in the CM equilibrium full-timers' welfare $S^*>0$, their joining probability $q^*$ must equal 1 and there must be at least some $t\in\{h,l\}$ such that the average earning rate $r_t^*>r_0$.*

*Proof of Lemma OA.8.* First, suppose instead full-timers' joining probability $q^*<1$. Their surplus $S^*=q^*M(u^*-u_0)>0$ implies that the expected utility on the platform $u^*$ is strictly higher than their labor market outside option $u_0$, which in turn implies more full-timers will choose to commit and thus the current probability $q^*$ cannot constitute an equilibrium. Contradiction.

Next, suppose instead the equilibrium average earning rate $r_t \le r_0$ for $t \in \{\mathsf{h}, \mathsf{l}\}$. Then for each committed full-timer the expected utility on the platform is $u^* = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (r_t^* - c_f)^* \le u_0 = r_0 - c_f$, which implies that $S^* = 0$. Contradiction. $\square$

LEMMA OA.9. *If in the CM equilibrium full-timers' welfare $S^* > 0$, their gross arrival rate $\lambda_f$ must be strictly less than $\mu_h$.*

*Proof of Lemma OA.9.* First, suppose instead full-timers' joining probability $q^* < 1$. Their surplus $S^* = q^* M(u^* - u_0) > 0$ implies that the expected utility on the platform $u^*$ is strictly higher than their labor market outside option $u_0$, which in turn implies more full-timers will choose to commit and thus the current probability $q^*$ cannot constitute an equilibrium. Contradiction.

Next, suppose instead the equilibrium average earning rate $r_t \le r_0$ for $t \in \{\mathsf{h}, \mathsf{l}\}$. Then for each committed full-timers the expected utility on the platform is $u^* = \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (r_t^* - c_f)^* \le u_0 = r_0 - c_f$, which implies that not all full-timers will commit, i.e., $q^* \le 1$. Contradiction. $\square$

LEMMA OA.10. *If $c_a \le r_0$, full-timers' welfare $S^*$ in the CM equilibrium equals 0.*

*Proof of Lemma OA.10.* Consider first the case with $c_a > c_f$. Suppose instead full-timers' welfare in the CM equilibrium can be positive, i.e., $S^* > 0$. In this case, Lemma OA.8 implies the existence of some period $t$ such that the equilibrium average earning rate $r_t^* > r_0$ and, given the condition $r_0 \ge c_a$, also $r_t > c_a > c_f$. But then full- and part-timers' participation probabilities $q_{f,t}^*$ and $q_{a,t}^*$ must both equal 1, and one can infer that the optimal piece-rate wage the company has chosen for period $t$ must be $w_t^* = r_t(1/(\mu_t - (q_{f,t}^* q^* \lambda_f + q_{a,t}^* \lambda_{a,t})) + 1/\tau)$. But then the company can make a strictly higher profit by instead paying workers $w_t^\epsilon = (r_t^* - \epsilon)(1/(\mu_t - (q_{f,t}^* q^* \lambda_f + q_{a,t}^* \lambda_{a,t})) + 1/\tau)$ for some small $\epsilon > 0$ such that $r_t^* - \epsilon > c_a$ and thus $q_{f,t}^* = q_{a,t}^* = 1$; that is, the company can lower the wage without reducing the number of workers who choose to participate. Contradiction. $\square$

LEMMA OA.11. *Define $\lambda_{E,t}^\dagger = \min\{\lambda_f, \mu_t - \sqrt{\mu_t \tau r_B/(p_t\tau - r_B)}\}$, the solution to $\max_{\lambda \le \min\{\lambda_f, \mu_t\}} \Pi_{E,t} \equiv (p_t - r_B(1/(\mu_t - \lambda) + 1/\tau))\lambda$ for $t \in \{h, l\}$. The company's optimal profit $\Pi_E^*$ in the EM is less than $\Pi_E^\dagger \equiv \sum_{t\in\{h,l\}} \beta_t (p_t - r_B(1/(\mu_t - \lambda_{E,t}^\dagger) + 1/\tau))\lambda_{E,t}^\dagger$ Furthermore, we have $\lambda_{E,h}^\dagger \ge \lambda_{E,l}^\dagger$, and the optimal workforce $\lambda_E^*$ in the EM satisfies $\lambda_E^* \in [\lambda_{E,l}^\dagger, \lambda_{E,h}^\dagger]$.*

*Proof of Lemma OA.11* We have $\Pi_E^* \le \Pi_E^\dagger$ since

$$\begin{aligned} \Pi_E^* &= \max_{\lambda \le \lambda_f, \lambda < \mu_{\mathsf{h}}, \lambda < \mu_{\mathsf{l}}} \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \beta_t (p_t - r_B(\frac{1}{\mu_t - \lambda} + \frac{1}{\tau}))\lambda \\ &\le \sum_{t\in\{\mathsf{h},\mathsf{l}\}} \max_{\lambda \le \lambda_f, \lambda < \mu_t} \beta_t (p_t - r_B(\frac{1}{\mu_t - \lambda} + \frac{1}{\tau}))\lambda = \Pi_E^\dagger. \end{aligned} \tag{OA.27}$$

Note that, the first-order condition for $\lambda^{\dagger}_{E,t}, t \in \{\mathsf{h}, \mathsf{l}\}$ is

$$p_t - r_B(\frac{\mu_t}{(\mu_t - \lambda)^2} + \frac{1}{\tau}) = 0, \tag{OA.28}$$

and for any $\mu > \lambda$

$$\frac{\partial \mu / (\mu - \lambda)^2}{\partial \mu} = \frac{-(\mu + \lambda)}{(\mu - \lambda)^3} < 0.$$

Since $\mu_{\mathsf{h}} > \mu_{\mathsf{l}}$ and $p_{\mathsf{h}} > p_{\mathsf{l}}$, plug $\lambda = \lambda^*_E$ into the RHS of (OA.28) for $t \in \{t, l\}$ and we have

$$p_{\mathsf{h}} - r_B(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \lambda^*_E)^2} + \frac{1}{\tau}) > p_{\mathsf{l}} - r_B(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - \lambda^*_E)^2} + \frac{1}{\tau}). \tag{OA.29}$$

Yet $\lambda^*_E$ is given by the first-order condition

$$\beta_{\mathsf{h}}(p_{\mathsf{h}} - r_B(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \lambda^*_E)^2} + \frac{1}{\tau})) + \beta_{\mathsf{l}}(p_{\mathsf{l}} - r_B(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - \lambda^*_E)^2} + \frac{1}{\tau})) = 0.$$

Combined with (OA.29), this implies that

$$p_{\mathsf{h}} - r_B(\frac{\mu_{\mathsf{h}}}{(\mu_{\mathsf{h}} - \lambda^*_E)^2} + \frac{1}{\tau}) > 0, \; p_{\mathsf{l}} - r_B(\frac{\mu_{\mathsf{l}}}{(\mu_{\mathsf{l}} - \lambda^*_E)^2} + \frac{1}{\tau}) < 0. \tag{OA.30}$$

Since for any $t \in \{\mathsf{h}, \mathsf{l}\}$, we have

$$\frac{\partial^2 \Pi_{E,t}}{\partial \lambda^2} = -r_B \frac{2\mu_t}{(\mu_t - \lambda)^3} < 0,$$

i.e., the company's profit is concave in $\lambda$, the results in (OA.30) imply that

$$\lambda^{\dagger}_{E,\mathsf{h}} \geq \lambda^*_E, \; \lambda^{\dagger}_{E,\mathsf{l}} \leq \lambda^*_E. \square$$